S. KOVALEVSKAYA
HER LIFE AND WORK

MEN OF RUSSIAN SCIENCE

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SOPHIA VASILYEVNA KOVALEVSKAYA

HER LIFE AND WORK

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INTRODUCTION

The outstanding Russian mathematician Sophia Kovalevskaya lived and worked in the second half of the 19th century.

History knows of only a few eminent women scholars prior to Kovalevskaya. Here is a brief enumeration of some of them:

In the 5th century of our era Hypatie (370-415 A.D.), daughter of Théon d'Alexandrie, was famed as a scholar. She lectured on philosophy and mathematics and wrote commentaries to the mathematical works of Apollonius and Diophante.

Marquise Emilie du Châtelet (1706-1749), of whom Voltaire wrote, translated Newton's *Principia* from Latin into French and compiled commentaries to it.

In 1748 the Italian Marie-Gaétano Agnesi (1718-1799) wrote *A Course on Analysis for the Use of the Italian Youth.*

Marie Lalande, who was born in Paris in 1768, assisted her brother, and later her husband, in compiling mathematical tables.

The Englishwoman Mary Somerville (1780-1872) wrote several books on physics and astronomy.

Sophie Germain (1776-1831) was highly gifted. In 1808 she was awarded the Napoleon Prize of the Paris Academy of Sciences for research on the theory of resilience.

Kovalevskaya's talents were outstanding. She lived in a century when Russia was making great strides in many

branches of knowledge: the brilliant chemist Dmitry Mendeleyev put forward his periodical system of elements, the basis of modern chemistry; Ivan Sechenov became famous for his research in physiology and the cerebral reflexes, while Alexander Stoletov laid the foundations of electronics.

Russian mathematicians also achieved great successes. At the turn of the 19th century the celebrated Lobachevsky originated a new geometry; Ostrogradsky and Bunyakovsky made valuable contributions to mathematical science; Academician Pafnuty Chebyshev was known the world over.

Sophia Kovalevskaya ranks on a level with these scholars.

The tsarist government did not appreciate Kovalevskaya's ability and she was forced to work abroad. But Kovalevskaya proved that women can master mathematics—the most abstract of sciences—and she set an example for many Russian women.

Kovalevskaya was not only a gifted scientist but also a talented writer. She presented a realistic picture of Russia during the sixties, seventies and eighties of the last century. Her *Childhood Reminiscences* was highly praised by contemporary critics. Two of her books—*The Woman Nihilist* and an unfinished story about Chernyshevsky—dealt with the activities of Russian revolutionary groups.

Her literary works are also valuable because they reflect the personal experiences of an outstanding Russian woman. Unfortunately, most of her writings remained uncompleted. Had she not died at such an early age, she undoubtedly would also have become eminent in the world of letters.

I. FAMILY BACKGROUND

Sophia Kovalevskaya (née Korvin-Krukovskaya) was born in Moscow on January 15, 1850. Her father, Vasily Krukovsky, an artillery officer, came of an ancient family ennobled for the military services of their ancestors—Ukrainian Cossacks. But it took him 15 years before his right to the privileged status of an ancient noble family was formally recognized. Krukovsky received the name of Korvin-Krukovsky because his family's coat-of-arms resembled that of the Polish family Korwin-Krukowski. He entered the army in 1817, and took part in the 1828 and 1829 campaigns in the Balkans against Turkey. He retired in 1858 with the rank of Lieutenant-General.

Krukovsky was a stern, reserved man. He was 40 years old when he married his young bride Yelizaveta Shubert. Well educated himself, he gave his children a good education, but did not approve of their progressive views. He was very angry when he learned that his elder daughter Anna had written a story for Dostoyevsky's magazine and had been paid for it. But gradually, under the influence of his daughters, he changed his outlook, and towards the end of his days became reconciled with their views.

Y. F. Litvinova, Kovalevskaya's first Russian biographer and a friend of the family, wrote that Sophia inherited her father's intellect and strong character.

Sophia's mother, Yelizaveta, was the daughter of a general, a geodesy specialist. She had received a good schooling and was well-read. Among her friends were Nikolai Pirogov, the famous surgeon, educationalist and public

figure; the Mathematics Professor Pyotr Lavrov, who was arrested by the tsarist government and then left the country, and the progressive artist Fyodor Moller, a friend of Gogol's. Yelizaveta liked to recall the evenings she had spent in their company discussing education, family life, etc.

Her acquaintance with Pirogov, who held progressive views on education, was of great interest. Pirogov advanced his views on education in an article published in 1856 entitled *Questions of Life*, in which, in particular, he wrote about the harm of special training without a broad background.

Although Yelizaveta was well versed in educational questions and had read Jean Jacques Rousseau, she paid little attention to the upbringing of her own children.

One day her two little daughters went to the woods by themselves and got lost. This incident, coupled with the elder daughter Anna's poor progress in her studies, brought home to Krukovsky that his children's upbringing was being neglected. He discharged their governess and engaged new teachers.

Sophia showed her aptitude very early. At the age of five she taught herself to read with the aid of the adults whom she would ask the meaning of this or that letter in a newspaper. She was put into the hands of expert teachers. But there was one sorrow that clouded her childhood and of which Kovalevskaya herself spoke. Her parents had wanted a boy when she was born, and they were consoled only five years later, when her brother Fedya was born.

Anna, the first-born, seven years Sophia's senior, was made a great fuss of. So also was Fedya. But Sophia got less attention, and she was jealous and suffered greatly.

She was extremely shy. But as she grew up she overcame this trait and became a lively, happy girl.

II. CHILDHOOD IN POLIBINO

When Korvin-Krukovsky retired from the army in 1858, the family went to live on their Polibino estate (formerly Palibino, Vitebsk Gubernia).

There Sophia spent her childhood, of which she later wrote in *Childhood Reminiscences.*

Polibino lies between Velikiye Luki and Nevel, to the right off the main road, about 17 miles from Velikiye Luki. The landscape has retained much of its beauty. The road rises and falls between the undulating spurs of the Valdai Hills; on both sides stretch endless fields, spotted here and there with young groves. There are many beautiful lakes in the region, and huge boulders—solitary sentinels of the ice age—are not infrequently to be seen. But the impenetrable pine forest which a century ago surrounded the estate is no more. Gone too are the packs of wolves which used to stage fearsome "concerts" in the vicinity of the estate in the winter. A birch-tree avenue leads to a beautiful old park. The avenue of limes where Sophia used to walk with her governess has grown very thick and shady.

The house was destroyed by the fascist invaders during the Second World War, and now only the shell-pierced thick walls remain.

In the summer-time, many tourists visit the estate where two outstanding Russian women spent their childhood and youth.

The house was a very massive affair, topped by a rectangular tower with a spire. This tower was the girls' favourite retreat, and it was here that Anna told Sophia about her first

steps in writing. Here, too, they dreamed of their future, of a better life for the people.

Their new teacher—Margaret Smith—was the daughter of English parents living in Russia. Miss Smith tried her best to transform the Russian girls into well-bred English girls. Anna, however, had already matured into a "free Cossack" and was not susceptible to foreign influence. Sophia was therefore the object of Miss Smith's full attention.

Sophia had to get up at seven o'clock. In winter it was still dark and she longed to sleep a little longer. Then came the cold shower which Kovalevskaya describes as "a breath-taking second of icy cold, followed by a sensation of boiling water throbbing through my veins; then I felt a wonderful surge of suppleness and energy spreading all through my body." After breakfast she had a music lesson at which Miss Smith kept time by tapping loudly with a cane. This was followed by an extremely dull 90-minute walk up and down the avenue with Miss Smith.

Sophia was glad when bad weather kept them indoors. There she was usually able to deceive Miss Smith and do a little reading in the library instead of playing in the adjoining corridor.

Sophia loved to write poetry and at the age of 12 she was convinced that she would become a poetess. She was especially proud of two poems: *A Bedouin Speaks to His Horse*, and *A Pearl Diver's Sensations*. One poem she devoted to the Panama Canal project which the press was widely commenting on at the time. Miss Smith, however, frowned on this poetry-making and tore up her verses whenever she had the chance. When Sophia misbehaved she would pin a paper to her back telling everyone of her offence.

But Miss Smith taught Sophia how to work systematically, and as the child was very painstaking and persistent she did not find it difficult to comply with her teacher's demands.

Sophia's first teacher on general subjects was Iosif Malevich, who was considered to be a very good tutor. Malevich began teaching Sophia in October 1858, when she was eight years old. "She was a sweet attractive child," he wrote in his memoirs, "of rather sturdy build, with hazel-brown eyes which sparkled with intelligence and kindliness." She was very attentive, quickly grasped a subject, punctually executed her tasks and knew her lessons well.

As Malevich's favourite subject was literature he at first was proud of Sophia's literary inclinations. He especially recollected her poem about the pearl diver.

"Surprised and delighted at her realistic, sensible, well-expressed point of view, which a philologist would have not been ashamed of," he wrote, "I would return to my room after a lesson and ponder for hours, not so much on the extraordinary success of my capable pupil, as on the future of a girl of a rich and esteemed family. What if she were ordained to traverse a different path in life? What would happen if fortune deprived her of excessive means for a livelihood and only provided her with moderate means for a higher education, which, unfortunately, women cannot receive in our universities? Then, oh yes! then I was positive that my talented pupil could occupy a prominent position in the literary world."

Malevich wrote that Sophia's progress in arithmetic was not unusual. One day at dinner her father asked her whether she liked arithmetic and she replied that she did not. But four months later she said she did like arithmetic.

Some four years of studies passed by smoothly, and Sophia made good progress. When she learned about the ratio of the diameter of a circle to its circumference she substituted her own solution for the one Malevich had explained. Malevich saw that she was correct, but insisted that she use his method, admonishing her for solving problems in a roundabout way. He wrote: "I don't know

whether she was taken aback by my unexpected demand, or I may have pricked her pride, but she flushed, cast down her eyes, and burst into tears."

When Malevich told Sophia's father about this incident the latter praised her, saying: "Good girl, Sophia. You're not like me. I used to be very glad to know my lesson anyhow and you, a slip of a girl, propose your own solution!" It was very gratifying for Sophia to be encouraged by her father.

Sophia studied arithmetic for two and a half years, then algebra and geometry for three and a half years, finishing with plane and solid geometry.

Kovalevskaya writes that it was her uncle Pyotr Korvin-Krukovsky who aroused her interest in mathematics.

Pyotr Korvin-Krukovsky was a tall, grey-haired man, somewhat eccentric. He had transferred his estate to his son and lived on a small pension. After the death of his wife, Sophia's uncle frequently visited Polibino. He was well-read, and Sophia,who was his favourite, liked to listen to his stories. There were heated discussions whenever he was present.

"Although he had never studied mathematics," wrote Kovalevskaya, "he had the greatest respect for it. He had gathered a smattering of mathematics from different sources and liked to philosophize on the subject, often expressing his thoughts in my presence. It was from him that I first learned of squaring the circle, of the asymptote—the line that approaches nearer and nearer to a curve but does not meet it—and about many other things of which I had then, naturally, not the slightest idea, but which excited my imagination, imbuing me with a reverence for this exalted and mysterious science that opened up wonderful vistas to the scholar, but was inaccessible to the layman."

Kovalevskaya relates an incident at Polibino which enhanced this interest:

"When we removed to our country estate, the house had to undergo major repairs and all the rooms had

to be papered. But we had many rooms and there was not sufficient wall-paper for one of the children's rooms. As we got our wall-paper from St. Petersburg it was not worth while ordering a small amount, thus the slighted room remained for a number of years pasted with sheets of paper which, by happy coincidence, turned out to be Ostrogradsky's lectures on differential and integral calculus which my father had bought in his student days.

"These sheets with their strange formulas soon drew my attention. I used to spend hours on end before the mysterious walls, trying at least to decipher separate phrases and find out the proper order of the sheets. By daily scrutiny the formulas and even the wording became imprinted on my brain, although at the time they were Greek to me."

Sophia relates how she studied mathematics with her cousin Michael, the only son of a widowed landowner. His mother wanted Michael to enter a secondary school, and he was being prepared for the 7th form. Michael, however, wanted to be an artist and considered all this swotting unnecessary.

He came with his mother to Polibino for one summer and Malevich began teaching him. Michael, however, was very insolent. He used to interrupt the lessons with all kinds of nonsense. He had to be made to study.

Sophia's help was enlisted. She was to show that even a girl who was younger than he could easily learn things. The ruse succeeded. Michael became ashamed of himself, and covered an extensive course of algebra and geometry that summer.

Nikolai Tyrtov (1822-1888), teacher of physics at the Naval School, presented his friend Korvin-Krukovsky with a copy of his *Rudiments of Physics*.

When he visited the Korvin-Krukovskys, he was surprised to learn that fourteen-year-old Sophia was studying his text-book independently. In the chapter on optics she came across the concept of the sine which hitherto she had

not known. She began to ponder on the mysterious formulas and unravelled their meaning. Tyrtov was amazed when Sophia explained to him the simple theorems of trigonometry. He called her a new Pascal and advised Korvin-Krukovsky to have her taught higher mathematics.

At a family council it was decided that Sophia would study mathematics during her visits to St. Petersburg with her mother and sister.

III. NEW POLITICAL TRENDS

In 1866 the two sisters and their mother went to Switzerland because of Anna's poor health. They stopped at Stuttgart to visit their cousin Sophia Adelung (who later became a children's writer and translator from Russian into German).

The two Sophias were of an age. They used to meet at their grandfather's house in St. Petersburg and at his summer residence at Pavlovsk.

Sophia Adelung wrote in her memoirs about the sisters, especially about Anna, whom she adored. Twenty-three-year-old Anna was blond, tall and slender, with large blue eyes and refined features. She had a good voice and loved to sing sad Russian songs, her favourite one being Lermontov's *Cradle Song*. Adelung, however, was mainly attracted by her cousin's gift of oratory. Usually reserved, Anna could be very eloquent when politics were being discussed. She was an ardent champion of reforms in Russia and of advanced political and social views. Her ideas were well-formulated and her plans clear-cut. She became furious at the mere word gendarme.

Adelung writes that although she was not versed in politics then, she believed in Anna as being the cleverest and most beautiful woman in the world.

To understand Anna's sentiments we must recollect the situation in the country at the time.

The revolutionary situation in Russia in 1859-1861 did not culminate in revolution, but gave birth to nihilism—a political trend, popular with the youth, which repudiated the

old society with its institutions, beliefs and morals. Kovalevskaya was a nihilist all her life and she dearly paid for this.

Sophia Kovalevskaya wrote that the sixties were very promising in Russia for political changes, for liberties and universal enlightenment.

The following excerpt from Kovalevskaya's writings shows how close she stood to the revolutionary youth circles:

"When several of us met for the first time at some private house where we could not express our views before our elders, a hint, a casual glance, a gesture was sufficient for us to understand one another, to know that we were of one camp. What great, secret, overwhelming joy, beyond understanding by others, rose in our hearts when we discovered that the young man or young woman whom we formerly, perhaps, had not met, with whom we had hardly exchanged a few remarks, but as we knew, was imbued with the same ideas, the same hope, the same readiness for sacrifice to attain the same goal as we ourselves."

In her *Childhood Reminiscences* Kovalevskaya describes the aims of this democratic movement of the sixties, writes of its champions—the inspirers of the youth—Chernyshevsky, Dobrolyubov, and Sleptsov. The Russian edition of the book omits these names, only the edition published abroad contains them. Chernyshevsky and Dobrolyubov were great Russian revolutionary democrats, philosophers of the materialist school, the forerunners of the Russian Social-Democrats. Maxim Gorky in an article about Sleptsov noted his wide range of activities, in particular the lectures he arranged for women on social and scientific questions, the "commune" he organized where women earned a livelihood by working as book-binders, seamstresses, copyists, etc.

Gorky wrote that Sleptsov was an acquaintance of Kovalevskaya's. This shows her close ties with the radical circles of the sixties.

The tsar's land reform of 1861 satisfied neither the peasants nor the progressive sections of Russian society. There was widespread discontent in the country and secret revolutionary bodies sprang up to fight Russian absolutism.

The two sisters at Polibino were well-informed about the political events. They read such journals as *Sovremyennik*, which had been established by Pushkin, and was later edited by Nekrasov and Chernyshevsky. Under the direction of the latter and with the participation of Dobrolyubov, Sleptsov and others this journal became the organ of the revolutionary democrats. They also read *Russkoye Slovo*, the journal of the intellectuals who did not belong to the gentry, and the prohibited *Kolokol* which Herzen published in London.

In April 1866, the revolutionary terrorist Karakozov made an attempt on the life of Alexander II. He was hanged in the autumn of the same year.

A wave of mass arrests followed. Among those arrested were Anna and Sophia's acquaintances—V. A. Sleptsov, P. L. Lavrov and others. *Sovremyennik* and *Russkoye Slovo* were banned for their "harmful" influence on the youth.

Adelung testifies that Sophia too was "electrified" by the progressive ideas in Russia: "She was ready to endure the test of fire, die the death of a martyr for her lofty ideals, for humanity."

Sophia could transmit her enthusiasm to others. She used to take part in conversations and carry away everybody with her spirit. Her large expressive eyes would probe the very soul of her interlocutor.

In 1866 and 1867, when the two sisters lived in Switzerland, a number of congresses, including the First and Second Congresses of the First International, were held in that country. They aroused broad press comment and were widely discussed in Russian émigré circles, especially as they were attended by Russian delegates. Naturally Anna and Sophia were influenced by these events.

Adelung wrote that Sophia's interests when she was about 17 were not limited to mathematics. She was of a versatile character and put her whole soul into everything that appealed to her intellect and heart: literature, the arts, natural science and social problems.

Despite the novelty of her trip she did not forget about mathematics. Sophia wrote home for books to be sent her with Malevich who was expected in Switzerland.

IV. IN ST. PETERSBURG

In the autumn of 1867, Sophia went to St. Petersburg with her mother. It was customary for landowners' families to spend part of the year in the capital (or provincial city), especially if daughters were to be brought out in society. But Sophia was not interested in balls—she was engrossed in mathematics and physics.

Kovalevskaya was taught in 1867 and 1868 by Alexander Strannolyubsky (1839-1903), teacher of mathematics at the Naval School. He was an ardent supporter of popular education and of education for women. He considered these two questions interconnected as it was precisely women who were called upon to teach selflessly in the schools. Strannolyubsky took part in the enlightenment movement of the sixties when public libraries, Sunday schools for adults, and other educational institutions were opened. The first Sunday school opened in 1859 in Kiev at the initiative of N. Pirogov. In the seventies the network of these schools grew and evening classes appeared.

The status of women was being debated at the time. In 1856 Pirogov wrote in his *Questions of Life* that girls should be educated as future mothers but should not be granted civil rights. Two years later M. N. Vernadskaya wrote that as long as labour was despised women would be subordinated to men, "because true freedom for women lies only in labour." Women started to struggle for their rights, for independence, for work. Many employment bureaus invited women to teach but they lacked adequate training. Secondary schools for girls were just beginning to open.

In 1860-1861 a liberal trend was initiated at St. Petersburg University. With special permission, Army men, teachers, writers, including women, were allowed to attend lectures (mainly on history and literature), without the right to take examinations or become graduates.

In 1862, following student demonstrations, the University was closed. When it opened the following year these "liberties" were abolished.

Advanced courses for women were opened in private homes in the sixties. One of these courses was opened by N. V. Stasova, sister of the famous art critic V. V. Stasov. Prominent scholars, such as the chemist Mendeleyev, the botanist Beketov and the physicist Usov, read lectures free of charge. Kovalevskaya attended these courses. Strannolyubsky taught algebra at the Alarchin courses which opened on April 1, 1869, and at other courses for women.

But women were also demanding higher education. In December 1867 the authoress Yevgenia Konradi appealed to the First Congress of Naturalists and Doctors to support the demand that women be allowed to enter the universities. The congress sympathized with this demand, but could do nothing practically.

A petition was then circulated by N. V. Stasova, M. V. Trubnikova and A. P. Filosofova addressed to the Government calling upon it to set up a women's university. Over 400 people, including Sophia and Anna, signed the petition during one week.

This great beginning was supported by Mendeleyev, Beketov, Bestuzhev, Sechenov and by many other prominent people, including the English philosopher John Stuart Mill, the French authoress Léonie Champseix (André Leo), who later distinguished herself in the Paris Commune.

Women felt that they were a force to be reckoned with, that progressive Russia was behind them. André Leo wrote to the initiators:

"You have conceived the most excellent thing in the world, and you will be credited with establishing in Russia an institution of which we here only dream. I sincerely wish you success and happiness."

But much water had to flow under the bridges of the Neva before a higher school for women was opened.

In 1878, however, the Bestuzhev Higher Courses for Women opened in St. Petersburg. It was financed mainly by private contributions, including those of many of its professors. Strannolyubsky headed its financial committee for 14 years, and Kovalevskaya was on its committee for a long time.

Kovalevskaya was delighted with Strannolyubsky and he was pleased with his pupil. At their first lesson on differential calculus he was surprised how quickly Sophia mastered the conception of the limit and the derivative, as if she had known them previously. Now the mathematical material stuck on the walls in the children's room in Polibino made sense.

Strannolyubsky approved of her intention of receiving a broad education (Sophia wanted to study mathematics, physiology, anatomy, physics and chemistry), and he advised her also to engage in practical work. Being interested in practical medicine, she gave medical assistance to the peasants in Polibino. The two sisters bred fishes and insects to study natural science.

* * *

Sophia went to Academician Chebyshev for advice on her mathematical studies. At that time Chebyshev (1821-1894) headed the Russian mathematical school and he occupied a prominent place in the scientific world. He was member of the Artillery Commission, of the Learned Council of the Ministry of Public Education, honorary member of many societies, and a member of a number of foreign Academies of Sciences. Later, as we shall see, he took a great

interest in Kovalevskaya's scientific work and encouraged her, but when she approached him then he could not even allow her to attend his own lectures because of the regime prevailing at St. Petersburg University.

The only road open to Kovalevskaya was the one traversed by other Russian women—to study at a Swiss university. The first Russian women doctors, N. P. Suslova and M. A. Bokova, studied at Zurich University. A whole host of women followed their example. One reason for the tsarist government opening the Medical Courses for Women in Russia was the fear of the penetration of foreign democratic ideas.

Sophia and Anna prepared to go abroad. Anna wanted to go to Paris to study the social and political life of the city. Other girls wished to join them. But it was evident that their parents would put up stiff resistance.

There were many cases at the time of young men freeing young women from their parents' guardianship by fictitious marriage. The "wife" was granted full freedom of action, and she would go abroad to study, with or without her "husband." Anna knew a person who was willing to assist them.

This person was Vladimir Kovalevsky. He was born on August 14, 1842. Kovalevsky visited London, where he met Herzen and for a time gave lessons to his daughter. In 1866 he reported Garibaldi's liberation campaign in Italy as war correspondent of *Peterburgskiye Vedomosti.*

In Russia Vladimir was engaged in translating and publishing foreign scientific papers, text-books and popular books on natural science. His brother Alexander Kovalevsky was an eminent zoologist. Several years after Vladimir met Sophia he made brilliant discoveries in the field of palaeontology.

Sophia created a deep impression on Vladimir. He wrote to his brother: "Despite her 18 years the Sparrow (Sophia's nickname.—*Ed.*) is extremely well educated. She speaks foreign languages as fluently as her own. She studies mainly

mathematics, is now tackling spheric trigonometry and integrals. She is as busy as a bee from morning till night and still is lively, sweet, and very pretty."

At first Kovalevsky intended to "marry" Anna, but when he met Sophia he resolutely declared that if he had to sacrifice himself at the altar of marriage it would be for the sake of science.

Sophia saw in Kovalevsky a man who shared her views. Vladimir wanted to drop his irksome publishing business and return to scientific work.

"Despite my experience of life," Vladimir wrote to his brother, "the number of books I have read, and my persistence, I cannot grasp and analyze political and economic questions half as quickly as she can; and she does not treat them lightly but profoundly.

"I think that our acquaintance will make a man of me, and I shall drop my publishing business and begin to study. I must say that she is a thousand times better, cleverer and more talented than myself. No words can describe her industry. I never imagined that a person could be so diligent.

"In short she is something remarkable, and I do not know why I deserve this luck."

Vladimir's publishing business was not a success, his debts were mounting up; sometimes he did not have the money to pay his translators and editors their salaries.

Of Sophia's plans we can judge from a letter she wrote in the summer of 1868.

"I prepare for my examinations and write my thesis. Anna puts her travelling notes into order. Then I study independently. Later on we form a colony and I go to Siberia. I encounter a multitude of difficulties, disappointments, but undoubtedly do some good. Anna writes a wonderful book, while I make a discovery. Then we open a mixed secondary school. I have my own physics laboratory, drop medicine and take up physics and mathematics applied to political economy and statistics."

Thus we see that Sophia and Vladimir intended to embark on a life full of intellectual interest. They were married in Polibino on September 15, 1868. Five years later, in 1873, they became man and wife.

* * *

After their marriage the Kovalevskys went to St. Petersburg. There they attended Sechenov's lectures on physiology and other lectures at the Medical and Surgical Academy.

Sechenov at his own risk and responsibility proposed that Sophia Kovalevskaya attend his lectures and he himself conducted the practical course with the couple. In order to avoid inquisitive glances Sophia came to these lectures accompanied by the public figure Dr. Pyotr Bokov, Vladimir Kovalevsky and her uncle Pyotr.

Sophia was surprised at the attention Sechenov paid to her studies and was embarrassed at the amount of time he spent on their practical course. But Sechenov sensed in her a future scientist, and as a supporter of education for women he tried to assist this talented woman.

Kovalevskaya's interests were diverse but she intended concentrating chiefly on mathematics, and now she could, thanks to Strannolyubsky, successfully study at the University.

Sophia's intensive studies, however, began to undermine her health, and she saw that her dream of a versatile education was unrealistic. At the end of 1868 she wrote to her sister:

"I am studying quite a lot, mainly the subjects I learned at Polibino, i.e., mathematics. You know, matchless Anna, I have almost made up my mind to drop medicine and enter the physics and mathematics faculty straightaway. Don't you think that will be better? I have now become convinced that neither medicine nor practical work is in my line. I am only happy when I am engrossed in my meditations, and if I do not concentrate on my beloved subjects now, in my best

years, I may miss an opportunity that will never return. I have become convinced that one cannot learn everything, and one life is barely sufficient to accomplish what I can in my chosen field."

The Kovalevskys decided to go abroad. Sophia, to study higher mathematics, and Vladimir—geology and palaeontology.

V. HEIDELBERG

Sophia Kovalevskaya not only wanted to receive a higher education herself, but she also wanted to blaze the way to science for other women.

When she heard that Yulia Lermontova, daughter of the Chief of the Moscow Cadet Corps, had been refused admittance to the Petrovskaya Agricultural Academy, she invited Yulia to go abroad to study. Kovalevskaya made a special journey to Moscow to convince Yulia's parents, and Yulia herself, of the necessity of taking this "risky and fearful step."

Kovalevsky, Sophia and Anna left for Vienna on April 15, 1869.

The Kovalevskys were given permission to attend lectures on physics at the Vienna University, but Sophia wanted to go to Heidelberg University, which was famed for such prominent scholars as Helmholtz, Kirchoff, Bunsen, and others. In Heidelberg life was cheaper, too. Sophia had a yearly allowance of 1,000 rubles. Vladimir received an annual income of about 600 rubles from his small estate. Therefore they had to economize, especially as they also assisted others.

A large number of Russians studied abroad in the sixties, especially with the closing of St. Petersburg University in 1861-1862. Zurich University was the most popular university and the one in Heidelberg was second best.

It was not easy for Sophia to receive permission in Heidelberg to attend lectures. When she applied to Kirchoff, he referred her to the Prorector. After seeing the Prorector

Sophia had to speak to several professors. Then a special committee was set up which ruled that the professors themselves should decide whether Kovalevskaya could attend their lectures. And the professors decided in the affirmative.

At first Sophia attended nine lectures a week, then eleven, including eight on mathematics. She heard Koenigsberger and Du Bois-Reymond lecture on mathematics, Kirchoff on physics, Helmholtz on physiology. She also attended some practical courses.

Klement Timiryazev, the renowned Russian botanist, was studying at Heidelberg at the time. He wrote in his memoirs that he was constantly in the company of the Kovalevskys. He recollected the liberal beginnings in the sixties when women were allowed to attend St. Petersburg University and the warm reception the students accorded them, and he wrote of the reaction which stunted these first green shoots.

Vladimir studied geology in Heidelberg for some time and then travelled about Europe on geological research.

Anna soon left for Paris, the "will-ó-the-wisp of the revolution" as many revolutionary-minded people called it. There she began to work as a type-setter and joined the French revolutionary movement. When her parents found out what she was doing, they stopped her allowance and Sophia had to send her money.

Shortly afterwards Yulia Lermontova arrived in Heidelberg and Sophia assisted her to get permission to attend Professor Bunsen's lectures.

Lermontova wrote in her memoirs that Sophia immediately attracted attention with her aptitude and soon the whole town was talking about the wonderful Russian student. People would stop to look at her in the streets and mothers would point her out to their children as an exemplary student.

Once during a lecture someone made a mistake at the blackboard in solving a theorem and nobody could discover it. Sophia, after some hesitation, and greatly agitated,

went to the blackboard and corrected the mistake. She was very shy and did not mix with the students.

Lermontova recalled with pleasure their student days and their Sunday walks in the beautiful countryside around Heidelberg, on the steep banks of the Neckar.

She thus describes Kovalevskaya:

"Her outstanding abilities, her love for mathematics, her sweetness and modesty endeared her to all. There was something extremely attractive about her. The professors thought the world of her; she was very industrious and could sit for hours on end solving some mathematical problem.

"Sophia's moral qualities were supplemented by such deep and complex spiritual qualities as I have never encountered in any other person.

"She was small, thin, with a roundish face. Her hair, of a dark chestnut colour, was closely cropped. Her face was highly expressive and mobile. The look in her eyes was constantly changing, now shining and sparkling, now deeply preoccupied. She presented a unique mixture of childish naïveté and great strength of mind.

"She conquered the hearts of all, men and women, young and old, with her unaffected charm and complete lack of coquetry. Sophia seemed to be totally unaware of the interest she aroused. She did not pay the slightest attention to her outward appearance, dressed simply, and was even a little untidy—a trait she retained all her life."

Along with Sophia, Yulia Lermontova received her Doctor's degree in 1874. Upon returning to Russia, Lermontova at first collaborated with A. M. Butlerov in St. Petersburg, and then with Professor V. V. Markovnikov in Moscow. But at the beginning of the eighties, the first Russian woman chemist dropped chemistry and went to live on her Semyonkovo estate, near Moscow, devoting herself to agriculture.

Yulia Lermontova (1846-1919) was Kovalevskaya's lifelong friend. She took care of Sophia's daughter Fufa during part of her friend's life, and brought her up after Kovalevskaya died.

Sophia assisted two other young women—Anna Yevreinova and Natalya Armfeldt—to get a higher education. For a short while the four girls lived together in Heidelberg, forming a little colony.

Anna Yevreinova (1844-1919) was Lermontova's cousin and a general's daughter. Anxious to receive a legal education, in November 1869, she ran away from home, and crossed the Russian frontier illegally.

After a brief period at Heidelberg, Yevreinova studied at Leipzig University. In 1873 she received her Doctor's degree with distinctions and became Russia's first woman lawyer. She wrote a number of articles devoted mainly to women's rights.

Natalya Armfeldt (1848-1887) was Sophia's cousin and a professor's daughter. She left home to study mathematics but became a revolutionary instead. In 1873 she joined the Moscow Chaika group.* Armfeldt was arrested several times, and in 1879 was exiled to a notorious prison for revolutionaries on the Kara River in Chita Region (formerly Transbaikal Region). Here she cared for the sick and helped the needy without regard for herself. She died of tuberculosis in 1887.

Sophia was the guiding star of Yelizaveta Litvinova (1845-1919). A. N. Strannolyubsky coached Litvinova for the secondary school examinations, and in 1872, after passing them, she left for Zurich to study mathematics.

The Zurich University and Polytechnical Institute were the only higher schools which accepted women students and awarded them diplomas. In 1873, however, the tsarist government ordered the Russian women studying in Zurich to return home lest they become infected with revolutionary ideas and threatened those who did not return with deprivation of civil rights.

Many of these women intellectuals took up the struggle

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* Chaika groups consisted of young revolutionary Narodniks who engaged in propaganda among the peasants.—*Ed.*

against tsarism upon returning to Russia. They joined Chaika groups and as weavers, doctors' assistants, teachers, etc., carried on revolutionary propaganda among the people.

Y. F. Litvinova did not comply with the tsar's ukase to return home. In 1876 she graduated from the Zurich University, and in 1877, upon presenting her thesis on the theory of the functions of the complex variable quantity, received the degree of Doctor of Sciences (Mathematics, Philosophy and Mineralogy).

When she returned to Russia in 1878, she found it extremely difficult to find work and later was deprived of her right to a pension. She was refused permission to take her Magister examinations and could not lecture at the Bestuzhev Higher Courses.

Lenin's wife, N. K. Krupskaya (1869-1938), thought highly of Litvinova, who had taught her. She wrote: "When Lenin said in 1921 during the discussion on the trade unions that the primary school must teach the children to think logically, I recollected how Litvinova taught us this.... Nobody told us that we were being taught to think logically, but we were."

The Bestuzhev Courses were closed (between 1886 and 1889), when Krupskaya graduated from secondary school. The official reason given was "reorganization," but Krupskaya wrote that it was closed "at the order of the tsarina, who considered that women should not study but stay at home and look after their husbands and children." When the courses opened again in 1889, Krupskaya entered the mathematics department.

As Litvinova could not engage in scientific work she took to writing. She wrote over 100 articles on education, ten biographies, including those of the mathematicians Lobachevsky, Kovalevskaya, Euler, Laplace, d'Alembert, the philosophers Aristotle, Bacon, Locke, the astronomer Struve and the French public figure Condorcet.

These were the first Russian women scholars to follow in Kovalevskaya's footsteps.

Sophia attended lectures at Heidelberg University in 1869-70. She studied a great deal, because she intended taking her examinations for a Doctor's degree in mathematics and mechanics.

The Kovalevskys spent the autumn of 1869 in London, where Vladimir among other things hoped to meet Charles Darwin. Here Sophia became acquainted with George Eliot; at her home she met Herbert Spencer, with whom she had a heated argument on women's rights. She wrote of these people in her memoirs.

In the spring of 1870, Kovalevskaya went to Paris where Anna was immersed in revolutionary activity. Anna took her sister to mass meetings, she heard Lefrance speak and met Lissagaray, Brousse, Malon and others who later played an active part in the Paris Commune. When Kovalevskaya returned to Paris in 1882, she renewed her acquaintance with them.

VI. STUDIES IN BERLIN

Sophia Kovalevskaya wished to attend the lectures of Karl Weierstrass (1815-1897), the famous Berlin Professor of Mathematics.

Weierstrass's parents were not wealthy. His father wanted him to study law, but Weierstrass became interested in mathematics. His funds ran out and he had to take his examinations without attending lectures. Then Weierstrass taught mathematics for a long time at a secondary school, until he attracted attention with his profound research work. He was offered a professorship at Berlin University.

It was characteristic of Weierstrass that he was very reluctant to publish the results of his research before he was absolutely certain that all his deductions were foolproof, and of this he never was quite certain.

Students from all over Europe flocked to hear him lecture on the theory of the functions of the complex variable quantity. Manuscripts of his lectures passed from hand to hand. Mathematicians wrote books on his theory, some without properly understanding them, others without even mentioning his name.

Sophia arrived in Berlin for the new term in August 1870. The University did not accept women students and its Learned Council did not make any exception for Kovalevskaya, although she was already known to mathematicians. She approached Weierstrass himself. He wrote to Koenigsberger about Sophia, and upon receiving a reassuring reply, suggested giving her private lessons.

The outbreak of the Franco-Prussian war considerably reduced the student body, and Weierstrass jokingly rebuked the Learned Council for refusing to replenish his audience with Kovalevskaya.

Sophia quickly became Weierstrass's favourite pupil. He repeated his university lectures to her, discussed his unpublished works, the latest scientific achievements, problems of stability, and new theories of geometry.

"Regarding Kovalevskaya's mathematical training," Weierstrass wrote to a friend, "I can testify that I have had very few pupils who can be compared with her for industry, ability, diligence and passion for science."

She studied with Weierstrass for four years (1870-1874). During her vacations she went home or visited Paris and Switzerland.

* * *

In 1871 the Kovalevskys visited Paris twice, because Sophia was very worried about Anna who took part in the Paris Commune. They were in the French capital from April 5 to May 12 (i.e., 35 out of the 72 days of the Paris Commune). They made this hazardous journey partly on foot and partly by boat on the Seine, running the risk of being shot. Both sisters nursed the wounded in the vicinity of the fighting.

Upon returning to Berlin, the Kovalevskys learned of the downfall of the Commune and of the shootings and reprisals taken against the Communards. The life of Anna's husband Victor Jaclard was in danger. They hurriedly returned to Paris and enlisted the assistance of their father Korvin-Krukovsky, who came to the French capital with his wife and helped to organize Jaclard's escape from prison.

* * *

After an illness in 1873, Sophia on her doctor's advice went to Switzerland. She stayed with Anna and Jaclard in Zurich, where they lived after leaving France.

Litvinova studied in Zurich at the time and in her memoirs she tells of Sophia's visit.

When she told the celebrated mathematician Schwartz, a pupil of Weierstrass's, about Kovalevskaya's expected arrival, he told her that he had already heard about this wonderful woman from his teacher. Sophia brought him Weierstrass's lectures on the theory of the Abel functions.

Upon becoming acquainted with Schwartz's work, Kovalevskaya expressed a desire to study with him, being interested in some problems that he was working on. When Schwartz told her that Weierstrass had been nominated Rector of Berlin University, she thought that he would not have time to study with her any more. But the perceptive Weierstrass, as if anticipating his pupil's thoughts, wrote her a letter dated August 20, 1873, stating that he would always find time for talks with her. Sophia was undecided whether to forsake her old teacher or not. She told Litvinova that although she had mastered Weierstrass's ideas and was attracted by the novelty of Schwartz's work, she would always be able to keep herself in hand and live where she should.

On the eve of her departure from Zurich, Kovalevskaya, together with Litvinova, attended one of Schwartz's lectures, where he presented her to his audience as an outstanding mathematician.

* * *

Sophia returned to Berlin towards the end of 1873, later than she had expected, because of an outbreak of cholera in the capital. Weierstrass now could not devote as much time to her as hitherto.

During her four years of study in Berlin Kovalevskaya not only covered the university course of mathematics but wrote three important works: 1) *On the Theory of Partial Differential Equations;* 2) *On the Reduction of Abelian Integrals of the Third Kind to Elliptic Integrals*; 3) *Supplementary Re-*

search and Observations on Laplace's Research on the Form of the Saturn Ring.

In 1874 Göttingen University conferred on Kovalevskaya the degree of Doctor of Philosophy with the highest distinctions, excusing her from the usual examinations.

Kovalevskaya's first work was of special importance, for it dealt with vital problems of mechanics and physics. The French mathematician Cauchy had examined similar problems but only in relation to more simple equations. Besides, Kovalevskaya discovered certain special instances which had never occurred to any of the mathematicians at those times. Kovalevskaya gave Cauchy's theorem a more polished and general form. Now this theorem is usually referred to as the Cauchy-Kovalevskaya theorem.

Kovalevskaya had to be well versed in the complicated Abelian theorem of integrals in order to solve the second problem.

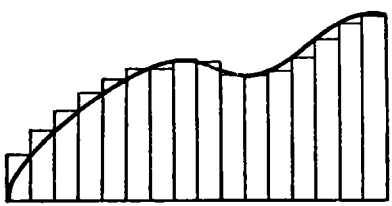


Fig. 1. Integrals determine the area of figures limited by curves.

Here is a simplified definition of the integral. An integral is the limit to which the sum of an infinitely large number of infinitely small items strives. Integrals are applied for calculating the area of figures limited by curves (fig. 1). The approximate aggregate area is found by dividing the figure into narrow strips, replacing the area of each strip by the area of a rectangle and adding these areas together. The more numerous the strips and the smaller the area of each strip, the more exact is the result. An absolutely correct estimate for the aggregate area is obtained by adding together the infinitely large number of infinitely narrow strips. Integrals are also applied for determining volumes, the length of arcs, etc.

The integral expressing the unknown area can be of varied complexity, depending on the nature of the boundary

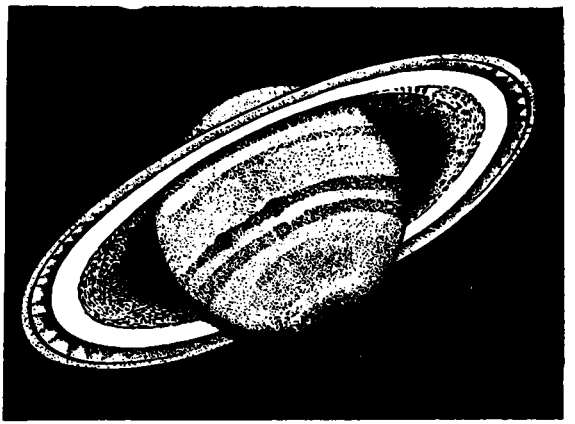


Fig. 2. Saturn encircled by a ring.

curve of the area. In many instances it falls under the category of Abelian integrals which are related to the Abel functions already mentioned. Abelian integrals are divided into

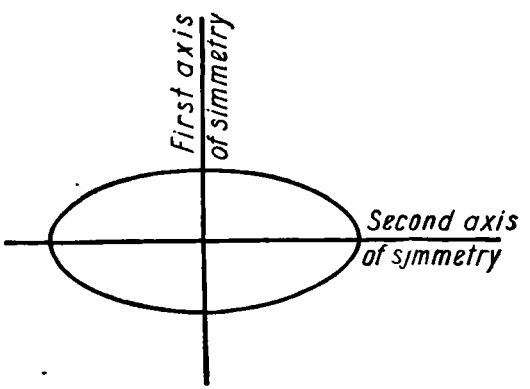


Fig. 3. Ellipse.

the first, second, third kinds, etc., according to their complexity. Weierstrass proposed that his pupil Koenigsberger simplify the Abelian integrals of the second kind. Kova-

levskaya solved a more difficult problem—that of the Abelian integrals of the third kind.

Sophia Kovalevskaya's third work was devoted to the study of the form of the Saturn ring (fig. 2). It was not known then whether the cross-section of this ring was round or limited by some flattened curve. The celebrated French scholar Laplace gave a simplified solution to this problem.

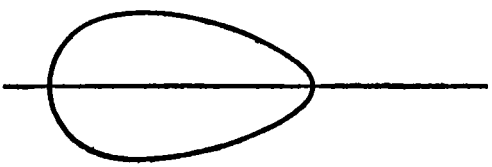


Fig. 4. Oval.

Assuming that the Saturn ring consisted of a number of thin rings, he established that its cross-section was bordered by an ellipse—a flattened circumference symmetric in relation to two mutually perpendicular axes (fig. 3).

Kovalevskaya found a more precise decision, proving that the cross-section of the Saturn ring must be egg-shaped, i.e., oval, symmetric only in relation to one straight line (fig. 4). Kovalevskaya's work was published in a German scientific journal, and later, with additions explaining her calculations, described in detail by the French astronomer, Tisserand, in his *Course of Celestial Mechanics.*

Another important problem which Sophia Kovalevskaya studied was that of the stability of motion. This can be illustrated by the behaviour of a spinning top, which, if hit, even sharply, will only temporarily divert from its original course. Stable motion is the form of motion when slight deviations (disturbances) caused by external forces disappear in time and the body reverts to its primary motion. If the slight primary deviation increases with the passage of time it is classified as unstable motion. Stable equilibrium is a specific

instance of stable motion. Stable equilibrium can be illustrated by the movement of a pendulum (fig. 5). When the pendulum is in its lower position, i.e., when its centre of gravity is beneath its point of suspension, its position is stable, for if we move the pendulum from position *a* it will return to its former position. In the pendulum's upper position, however, when its centre of gravity is above its point of suspension, its equilibrium is unstable, for the pendulum will easily deviate from its position.

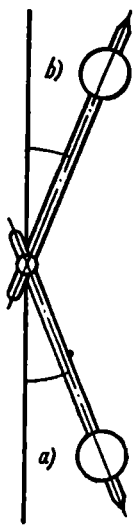


Fig. 5. Stable (*a*) and unstable (*b*) position of pendulum.

Of great cognitive interest is the problem of the stability of celestial bodies, which in certain stages of their development may consist of molten liquid mass. The forms which a liquid mass can assume while revolving uniformly with a

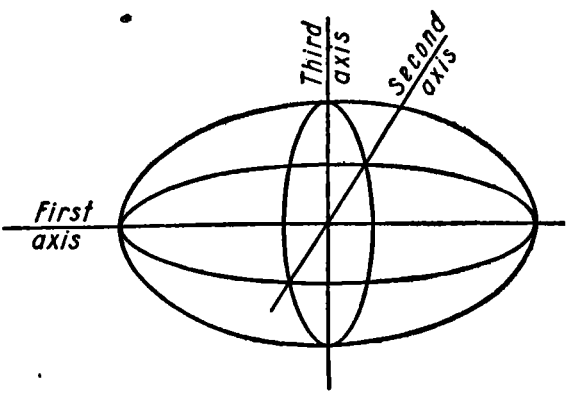


Fig. 6. Triaxial ellipsoid.

constant angular velocity are of great importance. It has been established that a triaxial ellipsoid (fig. 6) revolving around its short axis constitutes one of these forms. The liquid mass of a body revolving around an axis may also be ring-shaped (fig. 7) and pear-shaped (fig. 8). But pear-

shaped and ring-shaped celestial bodies are actually never encountered with (except the Saturn ring which we shall discuss later). This will be clear if we examine an important characteristic of revolving liquid—the degree of stability of its form. If particles of liquid after deformation resume their

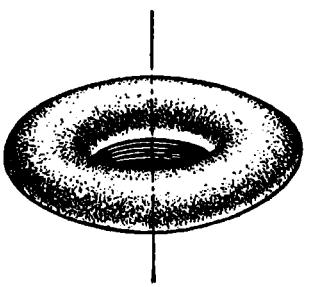


Fig. 7. Ring-shaped body.

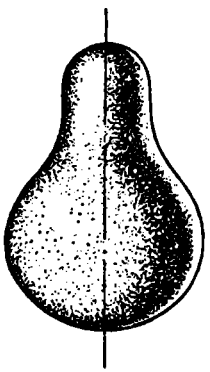


Fig. 8. Pear-shaped body.

primary form, their deformation is considered to be stable. If this deformation increases with the passage of time, causing disintegration of the body, it is considered to be unstable. Alexander Lyapunov and André Poincaré have proved that the stability or instability of a triaxial ellipsoid depends on the correlation existing between its size, mass and angular velocity of rotation (at very high speeds not only stability, but equilibrium is impossible).

Lyapunov established that pear-shaped bodies are unstable (i.e., that they can only exist if they are not subjected to any external influence, and the slightest disturbances may cause their destruction). This explains why no pear-shaped celestial bodies have ever been discovered. Kovalevskaya studied ring-shaped bodies and proved that they may be formed by revolving liquid of certain cross-sections. She

also examined the stability of ring-shaped bodies, but discontinued her research when it was established that the Saturn ring consists of separate solid bodies and not of liquid.

The instability of ring-shaped liquid bodies has been proved since Kovalevskaya's death. Hence the problem of ring-shaped and pear-shaped celestial bodies has been conclusively solved, and part of the credit for this is due to Sophia Kovalevskaya.

VII. HOME AGAIN

When the Kovalevskys returned to Russia, they planned to live and work in St. Petersburg, where they had numerous friends.

Kovalevskaya's feelings upon returning home can be discerned from her introduction to *The Woman Nihilist.* She enjoyed her success as a scholar and liked meeting people.

The Kovalevskys met Mendeleyev and Sechenov, Butlerov and Chebyshev, Stoletov and Turgenev, Dostoyevsky and many other prominent men of science and letters.

Mendeleyev arranged a party in honour of Yulia Lermontova when she returned to Russia, and he also gave one in honour of Kovalevskaya.

"Sophia immediately became the focal point of one of the élite circles, fervently devoted to intellectual interests, characteristic of the Russian capital and which is rarely met with anywhere else in Europe," wrote Anna-Charlotte Leffler.

They were also acquainted with political figures. Panteleyev in his *Memoirs* wrote that when he returned from exile in 1874, he was cordially received by the Kovalevskys. He relates that before he set out for exile in Siberia, from Vilnius in 1865, Vladimir gave him some money from his friends and even wanted to organize his escape. Sophia helped to provide legal aid for him and his friends and assisted their families with money.

The Kovalevskys' hopes of conducting scientific work in St. Petersburg, however, did not materialize. The scholars of the capital did not appreciate Vladimir's work and some considered him to be a dangerous competitor.

Despite her brilliant training, Sophia could not apply her knowledge in her native land. Women teachers could only teach arithmetic in the junior grades of girls' schools.

She wanted to teach at the Alarchin courses, but the students were insufficiently prepared for higher mathematics.

When the Bestuzhev Higher Courses for Women were opened in 1878, it was naturally assumed that Kovalevskaya would be invited to lecture, but she was not. This caused general surprise and indignation. Sophia was terribly disappointed. The reason was that she was considered to be a dangerous nihilist, closely connected with radically-minded circles.

In the 80s, when the government's policy became still more reactionary, Kovalevskaya's faint hopes of teaching at a higher educational establishment vanished completely, and she was obliged to turn her hand to journalism.

She wrote scientific and theatrical reviews for the liberal newspaper *Novoye Vremya*, and Vladimir, her husband, took part in publishing it. In 1877, when *Novoye Vremya* became reactionary, the Kovalevskys together with others resigned from its staff.

Kovalevskaya's articles were lively and interesting for she was well versed in her subject. She wrote about the latest achievements in science and technology, about optical apparatus, solar heaters, the telephone, the typewriter, about Pasteur's work on fermentation, about aviation and flying machines.

Her theatrical reviews were characteristic of her. She boldly criticized some foreign and Russian plays, including Ostrovsky's latest comedies. She would sometimes briefly dwell on how the actors played, would note the excellent acting of Strepetova or Savina, and admonish some bad actor. These reviews gave her material for writing on general subjects. She compared the acting of Russians with that of foreigners, noting the sincerity of the former, but criticizing them for insufficient plasticity in performing French plays.

Sophia is thought to have written a story in 1877 entitled

The Lecturer, about the life of German scholars, but unfortunately, the manuscript has been lost. In a letter to Anna-Charlotte Leffler in 1888, she wrote that she wanted to rewrite her first work, *The Lecturer*. The letter in part reads:

"I think that if it is completely recast I can make it into something splendid. I really am a little proud that when I was so young I understood some sides of human life so well. When I analyze E's feelings towards G I think that I have depicted the relations between my lecturer and his professor rather well. And what a splendid opportunity this can serve for preaching socialism! Or at least for developing the thesis that a pseudo-democratic, and *not* a socialist state represents the greatest horror that one can imagine."

Kovalevskaya's socialism was utopian, but nevertheless it would be interesting to know how she developed this thesis.

* * *

When the Kovalevskys returned from abroad they first lived at Polibino, where Sophia recuperated from the mental strain she had undergone. Anna, her husband and their son also lived at Polibino.

But the Polibino idyll could not last for long. They had to make a living.

The prospects of earning a livelihood by giving private lessons, by chance literary work or translations was not very attractive. The Kovalevskys wanted to earn enough to engage in scientific work.

In autumn 1875, Vladimir received a letter from his brother Alexander, a professor of zoology in Odessa, informing him that he intended buying a house, part of which he wanted to let.

Vladimir Kovalevsky had a brain-wave. He wrote to Alexander that an energetic person should not buy a house but build one. "This idea has just flashed through my mind," he wrote, "and I hasten to inform you of it. Undoubtedly it will entail a lot of bother, but Yazykov built a house and did not drop his legal business."

In the seventies Russia was experiencing an industrial boom. Some intellectuals, even scientists, went into business and industry, bought up real estate, became members of boards of companies, etc.

Vladimir had already shown his lack of ability in business, but Sophia attributed his failures to mischance. She defended him from his brother's scepticism and naïvely thought that she could help him.

In the same autumn Korvin-Krukovsky died, leaving his estate to his son and his money to his daughters. Sophia was deprived of the small but regular income she had received from her father, but she inherited 30,000 rubles, which, eventually, the Kovalevskys squandered on different undertakings.

They had a short period of prosperity which outsiders thought excessive. They dreamed of opening an advanced school for women. Various societies and public figures were opening such schools in different cities. But already in 1879 it became evident that their prosperity was a thing of the past. Their houses were mortgaged several times over, and their income was less than the interest they had to pay. "Sophia and I often regret at not having been satisfied with the little we had instead of going into business," wrote Vladimir to his brother on August 15, 1879.

The Kovalevskys went bankrupt. Sophia wrote that their creditors were very surprised to see so few possessions when they came to assess their property.

Sophia bore the blow stoically and did her best to sustain her husband. She settled accounts with their creditors as best she could and was eager to start life anew, to engage in mathematics.

She took this decision in the autumn of 1878. After not having written to Weierstrass for three years, Kovalevskaya sought his advice. But her daughter's birth on October 17, 1878, and the prolonged illness which followed it prevented her from starting work anew.

The Sixth Congress of Naturalists and Doctors, convened

in St. Petersburg at the close of 1879, heard papers on mathematical problems, and Chebyshev proposed that Kovalevskaya also read one.

She seized at the chance and hurriedly prepared a paper on the Abelian integrals. It produced a good impression and Chebyshev congratulated her.

During the breaks between the sessions, the young mother enthusiastically told Chebyshev about her daughter's achievements in saying "Mama," about her recent illness, the sleepless night she had spent before the session and such-like matters.

Seven years later Chebyshev in a letter to Kovalevskaya recalled what she had told him about Sophia the Second, as he called Fufa.

In the spring of 1880, after their unfortunate incursion into real estate, the Kovalevskys moved to Moscow. Vladimir was promised a lectureship at Moscow University at the beginning of the new term, and Sophia wanted to take her Magister's degree. Despite the support of several professors, the Minister of Education, Saburov, refused to allow her to take the examination, stating that both the mother and her daughter "will grow old before women are admitted to the University."

Meanwhile, Vladimir had to find employment. In the summer of 1881 he accepted the directorship of the Ragozin Brothers' Russian Refineries Company, a post for which he was ill fit. In autumn 1881, he went abroad on the company's business.

Sophia decided to go to Berlin during her husband's absence to get Weierstrass's advice on her plan of work.

She was very disappointed at not having received permission to take her Magister's degree. "Well, it can't be helped," she wrote to Alexander, her brother-in-law. "Now that it is extremely imperative for me to solve as many mathematical problems as possible so as to uphold the reputation of our sex at least in this way, I have decided to take a great risk—to go to Berlin for a month or six weeks

and leave my daughter in the care of Yulia Lermontova and Maria Dmitrievna (Fufa's nurse.—*Ed.*)."

Kovalevskaya arrived in Berlin on October 31, 1880, and returned to St. Petersburg at the beginning of January, 1881. During her trip abroad she visited London together with Vladimir, and met George Eliot again.

The mathematical problem which Sophia tackled was one which highly interested her—that of the refraction of light in crystals. But the Kovalevskys were confronted with more trouble after their return to Russia.

On March 1, 1881, Alexander the Second was assassinated. This was followed by a wave of arrests, and persons suspected of nihilist sympathies were persecuted. Under such circumstances it was impossible for the Kovalevskys to carry on normal scientific work.

In the spring of 1881, Sophia left hurriedly for Berlin with her daughter and nurse, and Vladimir went to his brother in Odessa.

Kovalevskaya put her whole soul into her work. She wrote to her husband: "You are quite right in saying that not a single woman has achieved anything as yet, but this makes it all the more imperative for me, while I still have the energy and a little money, to place myself in conditions which will enable me to prove whether or not I have the brains to achieve something."

Life abroad did not run smoothly. Sophia was worried about her husband's financial affairs, and she tried to economize as best she could. She also suffered from being separated from her husband.

One day in Paris Fufa fell ill in a noisy, draughty hotel. Kovalevskaya was terrified; she had no money and knew none of the local doctors. When Fufa recovered she was sent to her uncle in Odessa who also had children.

Although Kovalevskaya's circumstances were unsuitable for scientific work, she tried to complete her research as soon as possible, as it held out the possibility of material security. "Now in my old age," she wrote to her brother-in-law

in 1882, "I have to re-live the life of a student with its sorrows and joys. Every evening I am feverish with expectation for news from you. Does Fufa speak of me? Ask her whether she wants to return to Paris to her Mama."

When Fufa returned to her mother that summer, her father was undergoing the tragedy of being slowly but surely ruined. He did not want to admit this even to his brother, hoping against hope that his affairs would somehow improve. Several months later he had no more money to send to his wife.

Vladimir had come to the end of his tether. The Ragozin Brothers' Company was fraudulent and he was threatened with court proceedings. On April 27, 1883, he committed suicide.

Kovalevskaya was prostrated at the news. She kept to her room for four days and refused food. On the fifth day she fell unconscious and her doctor at last was able to start treating her. On the sixth day she recovered consciousness, instinctively took paper and pencil and sought refuge in some mathematical computations.

Her friend Maria Zaleska, the Polish revolutionary, was a great comfort to her during these difficult days.

When she sufficiently recovered, she left for Berlin, and from there returned to Russia.

What Kovalevskaya experienced in those days can be gauged from two passages in her writings: in *Childhood Reminiscences*, where she describes the emotions of a person who unsuccessfully tries to find solace from grief in sleep, and in *The Woman Nihilist*, writing about a person convalescing from a serious illness.

Her wound never healed completely. Lermontova wrote that Sophia would often cry when they were alone, blaming herself for abandoning her husband when he most needed her.

When Kovalevskaya returned to Russia in August 1883, she took energetic steps to rehabilitate her husband She proved that he had been but a tool in the hands of the Ra-

gozin Brothers and had not profited from their speculations in any way.

At a session of the mathematical section of the Seventh Congress of Russian Naturalists and Doctors, held in Odessa from August 30 to September 9, 1883, she was given the opportunity to report on the results of her more than two years' research into the problem of the refraction of light in crystals.

There she met the founder of the theory of aviation, Nikolai Zhukovsky (1847-1921). Kovalevskaya produced a very favourable impression on him and later he recalled their animated talk and noted her wide range of interests.

Kovalevskaya kept a diary which tells of the scientific problems which interested her. Four diaries have been preserved, covering the years 1881-1883, 1884, 1885 and 1890.

Her diary for 1881 opens on January 1 with a list of scientific literature, headed by Alexeyev's *Monograph on the Teaching of Gravity and the Theory of Potentials*, followed by Mathieu's *Course on Mathematical Physics* in French, Faraday's *Lectures on the Various Forces in Nature*, in English; Umov's *Equations of the Motion of Energy*, etc. On another page mention is made of George Darwin's *Theory of Tides*.

A number of books listed deal with optics and the theory of resiliency—subjects connected with her work on the refraction of light in crystallic mediums. Some articles she noted deal with the fundamentals of physics (the nature of space, the theory of the ether), with William Thomson's popular vortex atom theory, etc.

Sophia reviewed some articles. About one article from a U.S. journal for 1882 entitled *Energy as a Measurable Quantity* she wrote: " A very interesting article which can be quoted in full." She was interested in the long-distance transmission of electric current and its velocity, the distribution of the world's coal reserves, etc. It is possible that Kovalevskaya intended to continue writing scientific reviews. A number of books listed exposed spiritualism.

Her scientific notes and observations on everyday life are intermingled with verse. She wrote down a comic song of Edinburgh University students which ridiculed the vortex atom theory. On September 21, 1883, in Moscow, she wrote some verses of a defeatist nature by Pleshcheyev and Zhadovskaya into her diary. For instance, some lines from Pleshcheyev's poems:

We started forth with lofty aspirations,
But no compassion could we find around.
Dead are the hopes and the ideals we cherished.
Like faded leaves that fall upon the ground.
Killed by remorseless autumn, they have perished.

And then:

What life's cold cruelty could not succeed in killing,
Myself I buried, like the worthless slave I am.

These lines apparently reflected Kovalevskaya's pessimistic moments.

In 1883 Professor Gösta Mittag-Leffler, a pupil of Weierstrass's, invited Sophia to lecture at Stockholm University. He thought very highly of Kovalevskaya. He had heard her speak in 1880 at the Sixth Congress of Naturalists and Doctors, and had met her in St. Petersburg on another occasion. Then Sophia had written asking him to allow a Russian girl, Pokrovskaya, to attend his lectures at Helsingfors University. When he received the chair of mathematics in the new Stockholm University in 1881, he invited Kovalevskaya to join it.

At first Sophia was invited to conduct a course without payment, following which Mittag-Leffler hoped to get her a professorship of mathematics for five years.

The question of payment was very important for Kovalevskaya as she was left without means. But she accepted this offer for it gave her the opportunity of engaging in the work she loved.

VIII. LECTURING AT STOCKHOLM UNIVERSITY

Sophia Kovalevskaya arrived in Stockholm on November 18, 1883.

Litvinova wrote about how lonely Kovalevskaya felt upon witnessing the shores of her native land recede. After Kronstadt only the endless expanses of the grey, cold and desolate Baltic could be seen. The crossing took 45 hours, the last three hours of which the boat sailed past the numerous forest-covered islands of the Baltic fiord. Then the panorama of Stockholm came into view with its pointed Gothic towers and the huge rectangular block of the royal castle.

Sophia was cordially welcomed by Mittag-Leffler, his wife Signe and his sister Anna-Charlotte. The Lefflers did their best to make Sophia feel at home in the strange city.

The newly opened Stockholm University, where Kovalevskaya was destined to lecture for many years, attracted the liberal-minded Swedish youth, and it was a rival of the old Uppsala University—the stronghold of conservative traditions and the Church.

Kovalevskaya made many new friends in Sweden, but she also found some enemies, especially at Uppsala.

Her arrival aroused great interest and the press commented widely on it. "Today we do not herald the arrival of some vulgar, insignificant prince of noble blood," one democratic newspaper wrote. "No, the Princess of Science, Madame Kovalevskaya, has honoured our city with her arrival. She is to be the first woman lecturer in all Sweden."

Mittag-Leffler prepared a whole plan of operations designed to get Kovalevskaya elected to a professorship. He reached a compromise with some conservative professors by promising them support for two of their candidates, in return for their support of Kovalevskaya. His plan was a brilliant success. Mittag-Leffler afterwards related how several professors understood the "enormity" of what they had done when it was already too late.

After Kovalevskaya had read a popular course of algebra, many professors, including the University Rector Albert Lindhagen, adopted a friendly attitude towards her.

One professor publicly stated that nowhere in Scandinavia was mathematics so well taught as at Stockholm University.

The students appreciated her lectures too. When she completed a course on partial differential equations in the spring of 1884, they presented her with a nicely-framed photo of their group and delivered an enthusiastic speech. Sophia was deeply moved.

On the completion of this course, Kovalevskaya was elected to a professorship for five years.

The Swedish writer August Strindberg, an opponent of women's emancipation, wrote an article attacking Kovalevskaya. She described how he made it perfectly clear how harmful and embarrassing a female professor of mathematics could be. "I personally find that in essence he is right," joked Sophia. But she objected to his allegation that she had been invited to join the chair only out of consideration for her sex. Actually, by inviting Sophia Kovalevskaya, Mittag-Leffler aimed at making his chair one of the best in the world. In this he was successful.

Sophia Kovalevskaya conducted 12 courses during the eight years she lectured at the University. In 1884 she read a course on partial differential equations. She lectured on different branches of higher mathematics, on the latest works of Weierstrass and Poincaré, etc. Usually she lectured twice a week.

When professors fell ill, Kovalevskaya acted as substitute. One day she wrote to Mittag-Leffler, who was indisposed: "The mathematical department should by rights be called a mathematical ward, for I am the only person on my feet."

Kovalevskaya was a gifted teacher. She knew what every student was capable of and tried to develop their capabilities. Her lectures were always interesting.

One student said of Kovalevskaya that she could probe the very soul of a student, but it was very reassuring to be under her gentle, confident gaze.

Several women students attended Kovalevskaya's lectures. One, a Finnish girl, later taught mathematics in the senior classes of a boys' secondary school.

Kovalevskaya was very popular in Stockholm. People used to call her "our Professor Sophia." It was quite fashionable for parents to name their newly-born girls Sophia.

She also took part in editing the journal *Acta Mathematica*, which Mittag-Leffler founded in 1882. He wanted foreign mathematicians to write for the journal so as to give it a more varied character, and he asked Sophia Kovalevskaya to enlist the services of both old and young mathematicians when she visited foreign countries during her vacation. "It would be very nice," he wrote to her, "if Chebyshev visited Stockholm. We would do our best to give him a good reception. When you see him next time don't forget to remind him of the promise he made in Berlin to write for the *Acta*."

Sophia translated two of Chebyshev's articles into French and they were published in *Acta Mathematica*.

Kovalevskaya's duty on the editorial board was to read the manuscripts. The Russian contributors sent her their articles directly.

Some compatriots sought Kovalevskaya's advice on mathematical matters. For instance, the Kazan mathematician Maximovich sent her his work on imaginary calculus for appraisal.

Spiritualism was much in vogue at the time. Mendeleyev, Lev Tolstoi and other prominent scholars and writers came out in opposition to it. Some exponents of spiritualism tried to use mathematical conceptions to prove the existence of "another world."

In a letter to Kovalevskaya, the mathematician Tsvet, "mounting the forgotten mathematical Pegasus," as he put it, ridiculed this utterly absurd theory. He called Sophia the most wise nymph Egeria (the famous counsellor of Numa Pompilius of Ancient Rome), adding that he appreciated and adored her not only for her mathematical value but also "for her graceful and beautiful being."

When a prize for mathematical work was inaugurated in Sweden, Mittag-Leffler invited Kovalevskaya to take part in drawing up the conditions for its award, and in nominating the jury, to include Weierstrass and Chebyshev.

In 1885 the professor of mechanics at Stockholm University died, thus creating a vacancy. Mittag-Leffler put forward Sophia Kovalevskaya's candidacy, and after considerable friction she was elected. Kovalevskaya herself was not very keen on getting this post and she jocularly wrote to a friend that she had become a "professor squared."

Mittag-Leffler also wanted to get her nominated Member of the Stockholm Academy of Sciences, but she objected, not because she felt inferior to any Swedish candidate, but because she did not wish to make any more enemies.

Thus we see that Kovalevskaya's life was full of scientific interests.

She spent her vacations either in Russia, Germany or France. In the summer of 1886 her sister Anna fell seriously ill and Sophia nursed her without regard for herself. Kovalevskaya asked Mittag-Leffler to extend her vacation, but he refused, because he did not want to give her opponents a pretext to attack her, as no man would receive leave for nursing a relative.

Anna died after an unsuccessful operation in Paris in the autumn of 1887. This was a severe blow to Sophia, who adored her "spiritual mother," as she regarded her sister since childhood.

* * *

Sophia Kovalevskaya continued to write in Sweden. She could never say which she preferred—mathematics or writing. When she tired of abstract reasoning, she involuntarily turned to observing life and writing about it, and vice versa. "Everything in life sometimes seems insignificant and uninteresting," she once wrote, "and only the eternal immutable scientific laws attract me." Kovalevskaya considered that she could have done more had she concentrated on one occupation, but she could drop neither mathematics nor writing.

She alternated scientific studies with writing. For instance, after she had written her study on the refraction of light in crystallic mediums, she wrote her reminiscences of George Eliot (who died in 1880).

This was a brilliant psychological essay written in connection with the publication of George Eliot's letters in 1884.

Later, we see how she alternated her work on the problem of the movement of a solid body with writing.

In 1887 Sophia, in conjunction with Anna-Charlotte Leffler, wrote a drama entitled *The Fight for Happiness*. The main idea and construction of the play, as well as the broad contours of the leading characters were Kovalevskaya's. Its heroine, Alice, resembles Kovalevskaya herself, with her dreams of a better life for the workers and their friendship with the intellectuals.

Sophia described the hero Karl Torell as "an excellent specialist" but "uninteresting and rather helpless in private life." She considered that the mathematicians Hermite and Kirchoff, and the famous engineer Werner Siemens belonged to this type of person.

The play exposes the conservatism and callousness of the aristocracy and bourgeoisie. In Sweden it was severely crit-

icized, but in Russia, where it was presented after Kovalevskaya's death, it was favourably received.

Sophia was well abreast of Russian and foreign literature. The Swedish authoress Ellen Key wrote that Kovalevskaya soon knew Swedish literature better than the Swedes themselves.

Sophia Kovalevskaya tried to popularize Russian literature in Sweden. She introduced one translator to the poems of Nikitin and Dobrolyubov, and she edited a Swedish translation of Dobrolyubov's poem *Dear Friend, I Am Dying.*

She intended to write an article about Lev Tolstoi but apparently failed to do so.

* * *

The eighties of the last century saw an intense fight in Sweden between the new and the obsolete. Democratic and atheistic ideas began to emerge after the dark reign of the Church, and people began to show more interest in literature, the arts, etc.

A literary club was established in Stockholm which was frequented by people of diverse political views. Kovalevskaya was elected a member. In protest against the admittance of a nihilist to club membership, the editor of the most widely-read Swedish newspaper resigned.

Sophia was a very popular member of the club. She was called the "Michelangelo of conversation" for her liveliness and wit.

Hostesses were proud to announce that the two celebrities—Kovalevskaya and the explorer Fridtjof Nansen—would be among their guests at a dinner party.

She was well acquainted with Nordenskjold, another polar explorer. He was a man of varied interests and displayed an interest in social problems. The explorer was not allowed to lecture at Helsingfors University because of his progressive views.

Kovalevskaya also knew Ibsen, Grieg, the Dutch critic George Brandes, and many other men of letters and the arts.

Her best friends were the Lefflers and the Gyldens.

Hugo Gylden, head of the Stockholm Observatory, had worked for nine years at the Pulkovo Observatory near St. Petersburg. Fufa liked to play with the Gylden children, and she lived with them for some time.

Sophia went in for sports. She derived little pleasure from horse-riding as she was afraid of horses, but enjoyed skating with Leffler and his sister. People joked that Leffler used to continue his mathematical discussions with her by tracing formulas on the ice.

Although Kovalevskaya had many good friends in Stockholm, she dreamed of returning home. Fufa was beginning to forget her Russian. She made spelling mistakes and spoke with an accent.

Kovalevskaya knew French, German, English and Swedish well, but could express her thoughts best in Russian.

Sophia corresponded with Russian mathematicians: with Chebyshev and Selivanov in St. Petersburg; Vasilyev and Maximovich in Kazan, with Starkov in Odessa. They kept her informed about scientific life in Russia.

Total strangers wrote to her. One wanted to get a copy of an article published in some foreign journal, another asked for her photograph, a third requested her to recommend him a doctor knowing Swedish drill, a fourth needed a collection of Swedish minerals, etc. And of course there were the inevitable letters from her husband's creditors, whom she continued to pay off until her last days.

IX. MATHEMATICAL RESEARCH

In 1888 Sophia Kovalevskaya achieved her greatest success in the study of the movement of a solid body round an immovable point. This study can be illustrated by an ordinary top (fig. 9). A spinning top revolving on its axis will trace a certain curve (fig. 10). The time it spins depends on the degree of friction of its sharp end against the surface. The motion of a top is most remarkable. If slightly hit from a definite position the angle its axis will assume can be calculated in advance. Its deflection will be brief and the top will quickly revert to its original position. This is called the stability of the top's motion.

The gyroscope and gyro-compass operate on the same principle. The gyro-compass is widely used in the navigation of aircraft and ships, and the gyroscope is used as a stabilizer to resist the rolling of a ship or airplane.

Kovalevskaya made a detailed study of the gyroscope and its action.

Professor Nikolai Mertsalov designed a device to illustrate Sophia Kovalevskaya's work. This device consists of a narrow cylinder with two firmly joined thin rings. Two more rings (fig. 11) allow the cylinder to revolve freely round two axes—vertical and horizontal. One spot on the cylinder is luminous. The gyroscope was started in a dark room and its movement photographed. The luminous spot traced an intricate curve. Figures 12a and 12b show curves evolved from different starting points of the cylinder's axis at different rotation velocities.

Kovalevskaya's task was to calculate this curve and establish the position of a given point at any time.

It is very difficult to solve this problem for a gyroscrope of any form and in any position. Therefore, specific instances are investigated.

Fig. 9. Top.

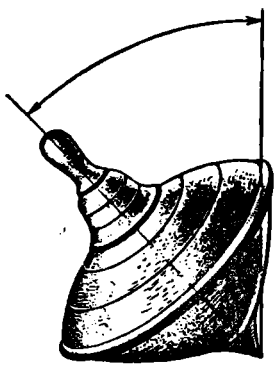


Fig. 10. Top.

Zhukovsky drew a chart depicting three specific instances of this movement (fig. 13).

The gyroscope in the first instance represents a body of arbitrary form, resting on a stand with its fulcrum coinciding with its centre of gravity. This case was examined by Academician Euler of the Russian Academy of Sciences.

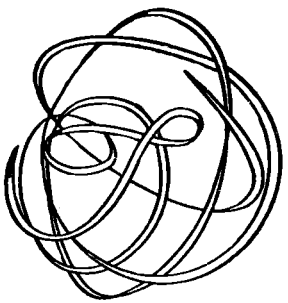
The second instance represents a revolving body produced by the revolution of some line round a straight line, i.e., its axis of symmetry; the centre of gravity of the body does not coincide with its fulcrum. Unless such a body revolves it will naturally collapse. However a *revolving* body may at a given moment take up the positions illustrated in fig. 12. This was the particular instance which the famous French mathematician Lagnange studied.

Sophia Kovalevskaya investigated a third instance of this motion by contemporary methods, which Zhukovsky depicted as a a third, slightly unsymmetrical top. In this case the centre of gravity is displaced in relation to its fulcrum.

This was a far more complicated problem than that in the two previous instances.

Thus, Sophia Kovalevskaya put herself on a level with Euler and Lagrange by her successful research into the classical problem of the movement of solid bodies. In order to accomplish this work she had to have profound knowledge of an intricate branch of mathematics—hyper-elliptic functions—which is related to the Abelian functions we have already mentioned.

The Paris Academy of Sciences had for a long time offered the Prix Bordin for "the perfection in one



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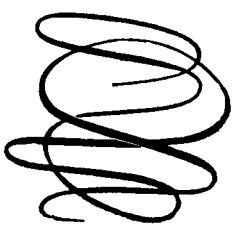

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Fig. 12.

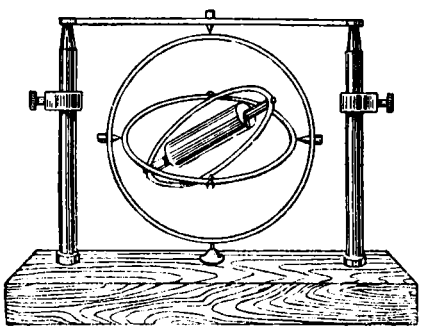

Fig. 11. N. Mertsalov's gyroscope.

point of the theory of the movement of a solid body round an immovable point." It had not been awarded owing to the absence of a sufficiently meritorious work.

In 1888 the Academy announced a new competition with the time limit fixed for the end of the summer.

That summer Kovalevskaya wrote to Mittag-Leffler: "I am so engrossed in mathematics that I can neither think nor speak of any other subject. I have achieved definite, and I must say, very pleasant results—that this instance of movement can really be integrated by ultra-elliptic functions.

But I still have to elaborate the conclusive formulas and I do not know whether I shall have time to do this by the end of the month. I cannot but tell you some details of my work. I shall be very brief because of insufficient time, but please try to understand the problem."

Sophia Kovalevskaya then set out the mathematical part of her work. About the time limit she adds: "This is very

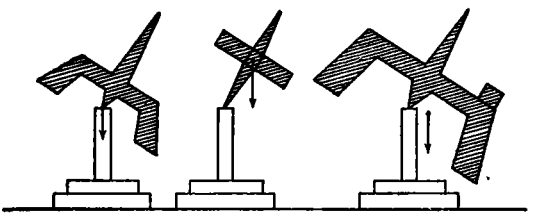

Fig. 13.

aggravating, because as you see my work has become rather interesting. The worst of it is that I am so tired, so worn out, that I sit and ponder for hours over a simple thing which at another time would take me half-an-hour to solve."

By a supreme effort Kovalevskaya finished her paper in time and submitted it with the dedication: "Say what you know, do what you must, come what may."

Sophia Kovalevskaya's paper was judged the best of 15 submitted and she was awarded the Prix Bordin with the prize money nearly doubled (from 3,000 francs to 5,000 francs) because of its importance.

There are five academies under the Institute of France, specializing in different branches of science. Private individuals contribute funds to these academies for the award of prizes for scientific research. Bordin, who endowed Kovalevskaya's prize, was a French lawyer who had bequeathed the academies a sum of money the annual interest of which amounted to 15,000 francs. This money was equally divided among the five academies. Bordin stipulated that the recipient of his prize must present a work of social significance which would benefit mankind, advance science and contribute to his country's glory.

Kovalevskaya received a letter from Louis Pasteur and Joseph Bertrand, the Permanent Secretaries of the Paris Academy of Sciences, dated December 18, 1888, in which they informed her of her award and the date on which she could receive it.

On December 24, 1888, Sophia Kovalevskaya, together with her compatriot of the same name Maxim Kovalevsky, were present at a special session of the Paris Academy of Sciences.

The President of the Academy, Pierre Janssen, in his speech of congratulation made special mention of Kovalevskaya's work: "Among the laurels which we present today," he said, "one of the most splendid and most difficult to attain will be laid on a woman's brow. Our co-members have found that her work bears witness not only to profound and broad knowledge, but to a mind of great inventiveness." The prize-winner was warmly applauded.

In 1889 the Swedish Academy of Sciences awarded her its prize for her further study into the movement of solid bodies. She published three articles on this problem.

The outstanding French mathematician Hermite, upon learning of Sophia's work, wrote to her that he awaited the publication of its results impatiently and hoped that he would "be able to pick up some of the crumbs from her table." He intended to study some of the integrals which Kovalevskaya had applied in her work.

Kovalevskaya studied the systems of non-linear differential equations similar to those met in the problem of the movement of a solid body.

Part of the results of her investigation she communicated to Mittag-Leffler in a letter, the rest of her results have been lost. A similar fate befell the results of her research into the general case of the movement of solid bodies about which she said she wrote to Hermite.

It is quite probable that as a result of her conversations with the French mathematicians she influenced the direction of their research in the field of differential equations.

Her research was the forerunner of a large number of studies on the same subject by such mathematicians as Goryachev, Chaplygin, Kolosov, Steklov, Nekrasov, Krylov and others.

Appelrot, who devoted his life to investigating this same problem, in 1940 compiled a voluminous study for a symposium published in memory of Kovalevskaya. His initial work analyzing the first part of Kovalevskaya's research was published shortly after her death. Appelrot was an admirer of Kovalevskaya's all his life, and he said that her work on the movement of a solid body "sparkled with talent."

Mathematicians were highly interested in this instance of movement. Zhukovsky worked out a geometric interpretation of Kovalevskaya's work. Delone studied specific instances of the movement of her gyroscope and designed a device to illustrate this movement. Mertsalov designed a device for the same purpose.

Mlodzeevsky and the Italian mathematician Levi-Civita also studied Kovalevskaya's gyroscope.

The problem of the movement of solid bodies of arbitrary form has not yet been solved completely. But whatever the result, Kovalevskaya's name will always be coupled with it.

* * *

Kovalevskaya was known to mathematicians prior to her discoveries. Some admirers wrote poems about her. In 1886 the English mathematician Sylvester wrote a sonnet comparing Kovalevskaya with the Muse of the Earth and calling her the Muse of the Heavens, whose star shines over Lake Malar, and who, with the might of Prometheus, compels the strings of the all-conquering mind of Man to resound.

Upon receiving the Prix Bordin, Kovalevskaya became famous. Newspapers and magazines devoted many articles to her. Sophia was certainly pleased with her success, but she dreamed of returning to Russia. Her professorship at the Stockholm University expired in 1889, and she had to take steps to renew it or find other work.

One day in Paris she met her cousin, Lieutenant-General Kosich, Governor of Saratov, whom she had not met for many years. He felt how she longed to return home, and so he wrote a letter to the President of the Russian Academy of Sciences, Grand Duke Konstantin, asking him to bring Sophia Kovalevskaya back to Russian science. He cited a saying, attributed to Napoleon, that "every state should assess the return of outstanding people to their country higher than the conquest of a rich city."

Veselovsky, the Academy's Permanent Secretary, sent Kosich the following reply:

"Sophia Kovalevskaya who has achieved fame abroad by her scientific work, is no less known to our mathematicians. The brilliant success of our compatriot abroad is all the more flattering to us because it must be solely attributed to her high qualities, as national sentiment could not serve to strengthen enthusiasm in her favour. It is especially complimentary to us that Madame Kovalevskaya received the post of Professor of Mathematics at Stockholm University. A woman could be elected to a university chair only as a result of an especially high and quite exceptional opinion of her ability and knowledge; Madame Kovalevskaya has fully justified this opinion by her splendid lectures. As chairs in our universities are closed to women, irrespective of their abilities and knowledge, there is no position for Madame Kovalevskaya so honoured and remunerative as the one she occupies in Stockholm. The position of a lecturer of mathematics at the Higher Courses for Women is far inferior to one at a university. In other girls' schools where women may teach only elementary mathematics is taught."

Progressive sections of Russian society were indignant at this injustice to Kovalevskaya. Russian mathematicians decided to mark the scientific achievements of their compatriot.

The honorary degree of Corresponding Member of the Russian Academy of Sciences was conferred on distin-

guished Russian scholars who did not reside in the capital and upon foreign scholars.

On October 24, 1889, Chebyshev, Imshenetsky and Bunyakovsky submitted the following application to the Physics and Mathematics Department of the Academy of Sciences:

"We, the undersigned, have the honour to propose Sophia Kovalevskaya, D. Sc. (Math.), Professor of Stockholm University, for election as a Corresponding Member of the Academy in mathematical sciences."

To confer this degree on a woman, it was necessary to amend the Charter of the Academy of Sciences.

On November 16, 1889, the Academy passed the amendment by 20 votes to six. On November 19, 1889, the Physics and Mathematics Department of the Academy elected Kovalevskaya a Corresponding Member by 14 votes to three. On December 2, 1889, a general meeting of the Academy approved Kovalevskaya's election.

Academician Chebyshev sent Kovalevskaya a telegram expressing his pleasure at the fulfilment of his cherished dream.

Weierstrass congratulated Sophia Kovalevskaya on the first recognition in Russia of her academic achievements.

When Sophia Kovalevskaya visited Russia during her vacation in the spring of 1890, she was enthusiastically welcomed at a session of the St. Petersburg City Council as the first woman Corresponding Member of the Russian Academy of Sciences.

Replying to the address of the Lord Mayor, Likhachov, Kovalevskaya expressed her satisfaction at the growth of public education. It should be noted, however, that the general level of education in Russia at the time was very low; only about one-tenth of the army recruits could read and write and the percentage of literate women was considerably lower. Probably Kovalevskaya was referring to the start made by various societies in opening schools.

Sophia attended the examinations at the Bestuzhev Higher Courses for Women. She was presented with a photograph

of the school building signed by 24 students and bearing the inscription: "In kind remembrance of our highly esteemed Sophia Kovalevskaya, from students of the Higher Courses for Women, who are sincerely grateful for your visit. St. Petersburg, May 27, 1890."

This visit shows the improbability of Kovalevskaya's refusing to teach at the Bestuzhev Courses. The fact that the girl students were weaker in mathematics than the young men would not have disheartened her. She had fought to set up these courses since the age of 18, had sat on their financial committee for many years and had always displayed great interest in their work. Then we have the evidence of Stasova, who wrote in her *Notes* that Sophia dreamed of lecturing at the Bestuzhev Courses. Hence Veselovsky's letter must be interpreted as a diplomatic refusal to allow her to lecture.

It was precisely in 1889 that several women who had just graduated from the Bestuzhev Courses began to teach at them. One was Vera Iosifovna Shiff. She taught analytic geometry and, later, wrote books on problems of analytic geometry and on the principles of analysis. At the same time Y. F. Litvinova, D. Sc., was refused a post, because 16 years earlier she had not complied with the tsar's order to return to Russia and had remained in Switzerland to complete her higher education.

This refusal to allow Litvinova and, especially, Kovalevskaya—the first two Russian women Doctors of Science—to teach mathematics at the Bestuzhev Courses reflects the monstrous stupidity of tsarist officialdom.

After unsuccessful attempts to find work in Russia, Kovalevskaya tried to obtain a professorship in a French advanced school for women, established in 1881. Despite the support of some eminent French mathematicians, she was unsuccessful. Therefore in 1889 she consented to having her professorship at Stockholm University prolonged.

X. LAST YEARS

In 1888 Sophia met Maxim Kovalevsky, a prominent Russian lawyer who had been discharged from Moscow University in 1887 for criticizing Russian constitutional law. Kovalevsky lectured and studied abroad, mainly in France.

Viktor Loren, an outstanding Swede, established a fund for the advancement of the social sciences. Kovalevskaya was on the fund committee, which invited specialists to lecture on sociology. Kovalevsky was invited to lecture in Stockholm and here he met Sophia.

Maxim Kovalevsky owned an estate in the Ukraine and a villa (Beaulieu) near Nice. Kovalevskaya used to spend her vacations at Beaulieu and they sometimes travelled together. When Kovalevsky came to Stockholm he usually presented Fufa with huge boxes of chocolate and life-size dolls. He himself was tall and he liked big things.

Ellen Key, the Swedish authoress, thus wrote of her impression of the Kovalevskys.

"I met Kovalevskaya each week for a number of years," she wrote, "but in essence I only saw her once. This was at a concert where Beethoven's Ninth Symphony was being played. Sophia, contrary to her habit, was elegantly dressed —in a black silk dress trimmed with lace. Next to her sat her compatriot, the object of her love. The exquisite strains of music floated around them. Sublime serenity was reflected on Sophia's usually nervous features. She sat as if transformed. She was in love and was carried away by the music to a land of wondrous visions.... I was destined to see a

similar expression on her face only once more—when she lay on her deathbed."

Sophia took great interest in Maxim Kovalevsky's work. She attended his lectures and discussed them at length with him. Kovalevsky enjoyed these discussions. He said that Sophia had the knack of getting to the core of things. To her he dedicated his book *The Origin of the Family and Private Property*, which Engels referred to several times in his work *The Origin of the Family, Private Property and the State.*

In her turn Sophia described her literary plans to him. In 1889 she told him and Professor Ivanyukov a number of episodes from her childhood. Both men found them very interesting and advised her to write a book about them. Thus her *Childhood Reminiscences* were born. The book was published abroad in 1889 under the title of *The Rayevsky Sisters.* In 1890 it was published in a Russian magazine with some parts, including the chapter dealing with Dostoyevsky's trial, cut out by the censor.

Her story *Memories of My Childhood During the Polish Uprising* was published in Russian only after the October Revolution. It describes an incident of the 1864 Polish uprising, which indicates fourteen-year-old Sophia's warm sympathy for the Poles, whose revolt was ruthlessly suppressed by Muravyov the Hangman.

Kovalevskaya's *Childhood Reminiscences* was translated into many foreign languages and it was widely discussed abroad. The press devoted many articles to it, considering it to contain one of the best descriptions of the life of a Russian landowner during the fifties and sixties of the last century.

Sophia received many letters from Russian women asking her to continue her reminiscences. Such was her intention, but, unfortunately, death intervened.

The decease of Saltykov-Shchedrin, the great Russian satirist, greatly affected Sophia Kovalevskaya. In 1889 her essay on his works was published in Sweden. This essay

could not be published in tsarist Russia, and it appeared in her native country only after the Revolution.

In 1890 Kovalevskaya started working on a novel about Chernyshevsky. Instead of real names she used fictitious ones, in order, as she put it, to have "more freedom in details" and so that "the philistines too could read it with emotion and interest."

This manuscript was thought lost, but now part of it—32 pages—has been discovered in the archives of the U.S.S.R. Academy of Sciences. Sophia Kovalevskaya writes about Chernyshevsky (Mikhail Gavrilovich Chernov), his wife Olga Sokratovna (Marusya), and of the principal figures of *Sovremyennik*—Nekrasov, Sleptsov and others. Kovalevskaya also writes of the suffragettes associated with the journal—Nadezhda Suslova, Korsini, etc.

The scene is set at a party arranged by the *Sovremyennik* staff to mark a new issue of the journal. This was to symbolize the highwater mark of Chernyshevsky's success, in contrast to his grim future.

Ellen Key stated that Sophia wrote about one-third of this novel, which she intended to call *The Nihilist.*

In the last chapter, Chernyshevsky, thanks to his novel *What Is To Be Done?,* becomes famous overnight. He is acclaimed at the party as the inspirer and leader of the youth. He returns home late, his wife is already asleep. He crosses to the window and looks down on the winking lights of sleeping St. Petersburg. Chernyshevsky enters into conversation with the huge, awesome city—the home of coercion, poverty, injustice and oppression; he is determined to conquer it, to infuse it with new life, to gradually win over its entire population, as he had won over its youth. Finally he leaves the window and intends to kiss his wife, wake her and tell her of his success. Suddenly a loud knocking is heard at the front door. Chernyshevsky goes to the door, and is confronted by gendarmes who have come to arrest him.

Kovalevskaya did not know Chernyshevsky personally. She was 11 years old when he was exiled to Siberia. But

she knew his friends and some members of his family. She advised his eldest son Alexander on his studies, was well acquainted with his cousin, Pypin, and with former members of the *Sovremyennik* staff, including Panayeva.

This manuscript confirms Kovalevskaya's close ties with the revolutionary democratic circles, and it shows that to her last days she remained faithful to the ideas of the great revolutionary and held him in great reverence.

One chapter of her novel *The Woman Nihilist*, which she wrote during the last years of her life, was published in a Swedish magazine. Maxim Kovalevsky assembled the remainder and published it abroad. The novel was published in Russia considerably later, being banned for a long time by the censor. It is of an autobiographical nature. The plot tells the story of a girl who joins an underground revolutionary group to further the interests of her people. She marries a revolutionary whom she hardly knows in order to share the privations of his exile.

In the summer of 1890 Kovalevskaya wrote an essay about a university in Sweden which she had visited in 1889, where peasant youths studied in the winter-time.

Kovalevskaya left a large number of unfinished writings. Of special interest is the beginning of one book entitled *A Story of the Riviera*, written in Nice in 1890-1891, telling of Maxim Kovalevsky, who is called Mikhail Mikhailovich Zvantsev.

For her, writing was not an idle hobby. It was a medium for expressing her attitude towards the most vital problems of her day; towards social and political trends, towards the liberation and enlightenment of her people, towards the position of the working class. She also made a study of the life of the intellectuals, their interests, morals and family life; she was keenly interested in questions of personal happiness.

* * *

Sophia Kovalevskaya's relations with Maxim did not always run smoothly. They sometimes quarrelled and this caused her much grief. But then they became reconciled and things returned to normal.

Their basic trouble was that they could not live together. For instance, in the summer recess of 1888 they both were occupied with different tasks. Maxim had to go to his Ukrainian estate on business, while Sophia had to complete her research work. He was displeased because Sophia could leave him for her studies at any moment. When Kovalevskaya became a Bordin Laureate, she remarked bitterly that a famous actress could be far more attractive to a man than a woman who studied till her eyes got swollen.

Then it was difficult for these Russian exiles, both celebrities in their respective fields, to live together permanently. Stockholm did not give Kovalevsky sufficient scope and he lived mainly in France. Probably he could have returned to Russia.

Anna-Charlotte Leffler wrote that Kovalevsky's condition for marriage was that Sophia drop her teaching and scientific work.

I do not think that he voiced this demand outright. However, if Sophia had married Kovalevsky she would have been forced to give up her post at Stockholm University, and as she was not able to get suitable work either in Russia or France, it would virtually have meant becoming a wife and nothing more. She had already held this status during the second period of her marriage with Vladimir until she found sufficient strength to engage in scientific work. Now that she was a professor, a Prix Bordin Laureate, and a Corresponding Member of the Russian Academy of Sciences, she simply could not become an ordinary housewife.

When Sophia went to Nice for her winter vacation in December 1890, there were reasons for doubts as to the reception she would receive, but everything went well and she did not even wish to return to Sweden. She wrote a letter to Fufa describing the beautiful garden view from

their veranda "blooming with roses, camellias and violets, and oranges ripening on the trees."

Mittag-Leffler wrote Kovalevskaya an encouraging letter from St. Petersburg, stating that there was still hope for her to receive work in Russia. Sophia asked him for leave to go to Russia after the winter half-term. Maxim intended going to Russia at the same time.

It was rumoured in Stockholm that the Kovalevskys would at last get married in the summer of 1891. The Swedes, especially the women, were keenly interested in her engagement. After Kovalevskaya's death much was written about her private and professional life. Some people held the opinion that scientific work for a woman was detrimental to married life, supporting the thesis of the authoress Anne de Stahl, who claimed that a woman's fame was the finale of her personal happiness. But the question of happy marriages for women scholars was worthy of some thought. The majority of objections against equality for women boiled down to the argument that a woman who does "a man's job" can be neither a good wife nor a good mother.

Many women in Stockholm wanted to know how Kovalevskaya fulfilled her maternal duties.Some rebuked her for not bringing Fufa with her when she first came to Stockholm.

Sophia Kovalevskaya always had her daughter's interests at heart. She did not bring Fufa to Stockholm until she herself was firmly established. Fufa had a good home with Lermontova in Russia, and it was inadvisable to bring the child to a strange city before she could be properly looked after.

Kovalevskaya considered it of the utmost importance to develop her child's will power. She recollected her own childhood days and used to say that had her talented sister and husband more will power they could have achieved more in life. Sophia Kovalevskaya loved her child, but at the same time was exacting. She would spend a lot of time with Fufa, reading Russian books to her and playing with her. She kept her daughter in the company of other children.

Fufa liked to play with the Gylden children and she stayed with the Gylden family for some time.

Fufa had many toys. When Sophia was awarded the Bordin Prize, Weierstrass's little son was positive that all the prize money would be spent on toys for Fufa.

Kovalevskaya was a loving mother and she fulfilled her duties to her child to the best of her ability.

* * *

Kovalevskaya returned to Sweden from her vacation via Berlin at the end of January 1891. In Berlin she visited some friends, who said that she was very animated and happy, full of plans for scientific and literary work.

But on her way to Stockholm, which she reached on February 4, she caught a severe chill. Upon her arrival she lectured at the University and then spent the evening at a dinner party at the Gyldens. Sophia was smartly dressed and in high spirits. All of a sudden she felt feverish and left shortly afterwards. She got into the wrong horse-tram and travelled an extra distance in the cold. The following day pneumonia was diagnosed.

On Saturday, February 8, Fufa wrote to her god-mother Lermontova that her mother had fallen ill, that Ellen Key had spent two nights with her, and that now there was to be a nurse.

"The doctors say," she wrote, "that there is no great danger, but Mama will have to keep her bed for a long while. And she is not to be worried or else she will get worse.... No steamer leaves for Russia before Wednesday. Today is Sunday, so I shall continue this letter again on that day....

"Monday. Mama is a little better. She perspired in the night and today she does not run such a high temperature.

"Tuesday. Dear Mama Yulia! Last evening Mama took morphine and I was not allowed into the room. Frau Gylden was with her until seven o'clock; when she left Mother told

her that she felt better and she was so composed. She felt far worse during the night. They sent for Frau Gylden. She came and woke me up. A little later Mama began to breathe more hoarsely and all of a sudden she stopped breathing. I did not even notice how it happened.

"I am now with the Gyldens. I very, very much want you to come quickly. I am so sad... Fufa."

Sophia Kovalevskaya died on February 10, 1891, at the age of 41, and at the height of her creative powers. Her funeral was attended by many people, including professors and students. Maxim was immediately informed of Sophia's illness, but by the time he arrived she was already dead. In a speech at her funeral he described Kovalevskaya as a representative of the new Russia, of the Russia that was destined to be a country of peace, social justice and extensive scientific development. He said that, thanks to her knowledge, talent, and integrity of character, Sophia Kovalevskaya would be forever the glory of her native country.

Mittag-Leffler, then Rector of Stockholm University, thanked Sophia Kovalevskaya for the profoundness and clarity with which she had guided the students' studies, and for the treasure of friendship she endowed upon all who were dear to her heart.

Wreaths and telegrams of condolence came from different parts of Russia—from St. Petersburg, Kharkov, Tbilisi, Saratov.

Stasova wrote in her *Notes* on February 11, 1891: "Kovalevskaya has died! What a misfortune! She was not appreciated at home!"

Fritz Leffler, Mittag-Leffler's brother, wrote a poem about Kovalevskaya and distributed copies of it to those present at the funeral. It ends with the lines:

While Saturn's rings still shine, while mortals breathe,
The world will ever remember your name,
Will honour it with never-dying fame,
And you have well deserved your laurel wreath.

A granite monument, designed by Sultanov, was erected on Kovalevskaya's grave with the money collected by Russian women.

Anna-Charlotte Leffler published her memoirs of Sophia Kovalevskaya a year after her death.

Leffler described Kovalevskaya as a representative of the Russian intelligentsia. She paid tribute to the versatility, open-mindedness and broad outlook of the truly enlightened and free-thinking Russians. Leffler noted their ability to grasp new ideas at once. With their mental vitality, she wrote, they "combine such enthusiasm, such faith in their ideals as we find in no other nation in Europe."

The Dutch publicist George Brandes wrote that Leffler's book was dedicated to a great woman whose spiritual and practical life had been far richer than that of Scandinavian women. The Russian Minister of the Interior, Durnovo, however, thought otherwise. He considered that too much attention was being paid to an ordinary nihilist, and he did his best to stop Leffler's book from being published.

Litvinova was the first person in Russia to write about Kovalevskaya. Later she wrote of the days she had spent with Sophia in Switzerland. Kovalevskaya's literary works were published in Russian and the drama *The Fight for Happiness* was also translated.

The second part of this drama *How It Could Have Been* was staged by the Korsh Theatre in Moscow at the suggestion of the actress Yavorskaya.

The play was warmly received by the Russian public. They were attracted by the plot, and especially by the novelty of the theme as the characters included workers who resisted their employers. This great interest was also connected with the deep sympathy felt for the authoress, a Russian woman who had died prematurely, far from her native land.

Kovalevskaya anniversaries were regularly marked by progressive Russian society before the Revolution.

The centenary of Kovalevskaya's birth in 1950 was widely celebrated in the Soviet Union. Special evening parties and meetings were organized, and the press commented extensively. The U.S.S.R. Academy of Sciences held a special session at which its President, Vavilov, delivered the opening address.

Talks were arranged in schools and institutions, some were addressed by Kovalevskaya's daughter, who in 1952 died at the age of 74. She had received an advanced medical education at the Lesgaft Courses. During her last years she translated foreign literature into Russian. She had untiringly deciphered and translated her mother's letters.

A meeting was held in Stockholm to mark the centenary. The Soviet Embassy and Swedish organizations laid wreaths on Kovalevskaya's grave. Swedish newspapers published articles by women who had known her. It was noted that only 50 years after Kovalevskaya did a second woman Doctor of Science appear in Sweden (Nanna Svartz).

CONCLUSION

Kovalevskaya's scientific life was brief. Part of the four years she studied with Weierstrass was devoted to research. Then, after six years, Sophia began her second period of creative activity lasting only ten years. But during this brief period she achieved much.

Sophia Kovalevskaya published nine works dealing with six problems of mathematics and mechanics. Three studies she devoted to research into the movement of a solid body round an immovable point; two articles dealt with the problem of the refraction of light in crystals. One major work dealt with the theorem of the solution of the system of differential equations with partial derivatives. Kovalevskaya wrote several articles on the shape of the Saturn ring and on the Abelian integrals. Lastly, she published a work on Brun's theorem, elaborating a better and simpler method.

After her death the Moscow Mathematical Society held a special commemoration session at which Stoletov, Zhukovsky and Nekrasov spoke about Kovalevskaya.

Stoletov gave a brief outline of her life and work, referring to his personal acquaintance with Sophia Kovalevskaya and her husband Vladimir whom he visited in Moscow and of whom he cherished the warmest recollections.

The famous Russian scholar Zhukovsky spoke of Kovalevskaya's work in the field of mechanics. He dwelt in detail on the problem of the movement of a solid body, a problem to which he himself (as we have already mentioned) had contributed. He said that Poincaré had told him in Paris in the summer of 1889 about Kovalevskaya's further re-

search into the complete solution of this problem for the general case. Zhukovsky ended with the words: "Unfortunately an untimely death dashed all these hopes and deprived us of a compatriot who had done much in contributing to the glory of the Russians."

Professor Nekrasov delivered a third report on Kovalevskaya's work in the branch of pure mathematics.

All these scholars praised the work of the great Russian scientist highly, recognizing her full equality with talented male mathematicians.

She made brilliant discoveries by penetrating deeply into the methods of mathematical research.

Foreign scientists also rendered homage to this glorious Russian woman. They acknowledged that she had surpassed her predecessors in talent and results and occupied a most outstanding place among contemporary mathematicians.

Kovalevskaya's work is of great importance today. Soviet scientists have studied her gyroscope for stability. Kazan mathematicians have published a number of works on the integration of equations in partial derivatives.

Two Russian women—Nadezhda Hernet and Lyubov Zapolskaya—followed in Kovalevskaya's footsteps. In 1902 Göttingen University awarded them Doctorates for mathematical research. Hernet afterwards taught at the St. Petersburg Advanced Courses for Women and Zapolskaya at the Moscow courses.

Mention must also be made of prominent women scholars of other countries.

Marja Sklodowska-Curie (1867-1934) was a most outstanding scholar in the exact sciences. She was twice awarded the Nobel Prize for research in radio-activity.

Emmy Noether (1882-1935), Professor of Göttingen University, was another prominent woman scientist. She was the founder of a new trend in higher algebra. H. Weyl in a funeral oration on her death compared her life—that of an unmarried woman entirely devoted to science—with Kovalevskaya's turbulent and difficult life.

Now there are many women in the Sóviet Union and in foreign countries engaged in scientific research, in particular in physics and mathematics. Yekaterina Naryshkina (1895-1940) carried out intricate research in the field of resilient vibration. No one is now surprised at women professors and at women receiving degrees and prizes. But Sophia Kovalevskaya will always be distinguished among women scholars not only for her scientific achievements, but for the prominent part she played in blazing the trail for women in all fields of endeavour. This popularity is enhanced by her personal qualities, her integrity of character, her versatility, and her literary talent.

Sophia Kovalevskaya will always occupy a prominent place in the history of the social and scientific progress of Russia. She is and always will be the pride of her native country.